The influence of sale announcement on the optimal policy of an inventory system with perishable items

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A R T I C L E   I N F O

Article history:
Received 15 March 2016
Accepted 14 April 2016
Available online 26 April 2016

Keywords:
Inventory control
Perishable items
Sale announcement
Single-period

A B S T R A C T

In this paper a single-period inventory system with a sale announcement for perishable items is investigated. In this problem setting, the perishable product has a deterministic expiry date and a demand with a probabilistic behavior during the period. When the expiry date of the product is approaching, a special sale announcement may alter the customers’ behavior and escalate the demand rate preventing huge loss of the expired products. Two model is developed to obtain the optimal order quantity of the product and the optimal time for sale announcement. The first model considers a static price dependent behavior of customers independent from the product’s expiry date, while in the second one the product’s demand rate after sale announcement is assumed as an increasing function of its remaining lifetime. Usefulness of the proposed models and the influence of sale announcement on total revenue is demonstrated using numerical examples. Finally, a comprehensive sensitivity analysis is conducted revealing the effect of different parameters of the models on the optimal policy.

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1. Introduction

Perishable items are often modeled as a single-period problem known as newsvendor problem which the product should be ordered at outset and be sold during the period (Krider and Weinberg, 2000). In the basic form of a single-period inventory model, the buyer decides about the optimal quantity of purchased product at outset of the period with a probabilistic demand. There are different versions of the single-period inventory problem in the literature. Moon and Silver (2000) and Abdel-Malek et al. (2004) studied the problem for the budget constrained multi-item case. Serel (2008) integrated inventory and pricing decision in the newsvendor problem. Ölzer et al. (2009) investigated value-at-risk constraint in the single-period problem. Chen and Chen (2010) proposed a multiple-item newsboy problem with constrained budget and a reservation policy. Qin and Kar (2013) developed a single-period inventory model considering uncertain environment. Kamburovski (2014) considered the distribution free version of the problem under the worst-case and best-case scenarios. Rossi et al. (2014) modeled a new method to estimate demand by combining inventory optimization and statistics. A price-setting newsvendor is investigated by Ye and Sun (In press) with strategic consumers to determine optimal selling price and stock quantity while this problem is studied by Rubio-Herrero et al. (2015) under mean-variance criteria. A single-period problem considering late season low inventory assortment effects is investigated by Khouja (In press). Recently, Carrizosa et al. (2016) developed a robust approach to maximize the revenue in worst-case with autoregressive demand for the newsvendor problem.

Sale announcement has been one of the most effective tools to motivate consumers. Also, this strategy is an accepted way for retailers when expiration date of their perishable goods is approaching (Donselaar et al., 2006). Nowadays, advancements in intelligent identification and information systems has made waste reduction easier than past (Pramatari and Theotokis, 2009). On the other hand, recent researches show that sale announcement may have an impact on customer trust in a negative way reducing goodwill of the brand (Xia et al., 2010). Especially for food products, the sale announcement may be interpreted as a sign of decreasing quality or a very close expiration date. From the consumers’ viewpoint, newly produced high quality products with a reasonable price are preferred. However, any price-cut will result to a trade-off problem between cost and quality of the product. From the retailer’s point of view, quality of the product is expressed by the precisely calculated expiration date and quality of the product before its expiry date is assumed perfect. These conflicting perspectives have made the timing a discount as a crucial problem where a too soon or too late price-cut leads to a suboptimal policy (Johnson et al., 2013).

There are several works considering a variety of predefined sales and discount policies in the literature of the inventory planning systems (Zhang, 2010; Chen and Ho, 2011; Theotokis...
et al., 2012; Tromp et al., 2012; Chen and Ho, 2013). Based on the author’s knowledge, despite the importance of the timing of sale announcement in a single-period problem, there is no previous research in the literature addressing this issue. The key contributions of this paper are: (1) Extension of a single-period inventory system to obtain the optimal time of sale announcement and optimal order quantity at outset of the period. (2) Extension of the proposed model considering that consumers demand behavior after announcement of sale is a function of remaining time to expiration date. (3) Sensitivity analysis and comparison of the proposed models with the basic single-period model using several numerical examples.

2. Proposed models

In this section two novel version of the single-period inventory model is extended with a distribution-free structure for demand rate. Then the solution approach is presented for two special cases, i.e., uniform and exponential distributions.

2.1. First model: expiry-date-independent demand

In this subsection, the newsboy model is extended supposing that the retailer will announce a special sale to motivate consumers to buy more. It is assumed that sale price is predefined and the average demand after the sale announcement is given independent from remaining time to expiry date. Let \( x_1 \) and \( x_2 \) denote the random variables of demand per time unit before and after sale announcement with expected values of \( E(x_1) \) and \( E(x_2) \), and probability distribution functions of \( f_{x_1}(x_1) \) and \( f_{x_2}(x_2) \), respectively. Let \( v_1 \) and \( v_2 \) denote the known unit purchasing price before and after sale announcement, respectively. Also let \( c, x \) and \( h \) denote unit purchasing price at outset of the period, unit shortage cost during the period and unit waste cost of redundant product at end of the period, respectively. The average total revenue of the basic newsvendor problem is as follows Abdel-Malek et al. (2004):

\[
E(Z) = (v_1 + h)E(x_1) - (c + h)R + (x + v_1 + h)\beta_1(R)
\]  

where the decision variable, \( R \), is the quantity of purchased product at outset of the period and \( \beta_1(R) \) is the average shortage during the period. The cumulative probability in the optimal value of \( R \) to maximize \( E(Z) \) can be expressed as follows:

\[
F_{X1}(R^*) = \frac{(x + v_1 - c)}{(x + v_1 + h)}
\]  

At the moment which retailer announces a special sale after a fraction \( r \) of the period’s outset, the average remaining inventory is as follows:

\[
R = R - rE(x_1) + \beta_1(R)
\]  

Considering \( R \) as the initial quantity of the product after sale announcement the average total revenue of the proposed news-vendor problem can be obtained as follows:

\[
ATR(r, R) := (v_1 + h)E(x_1) + (1 - r)(v_2 + h)E(x_2) - \left[ (c + h)R + (x + v_2 + h)\beta_1(R) + (x + v_2 + h)\beta_2(R - rE(x_1) + \beta_1(R)) \right]
\]  

2.2. Second model: expiry-date-dependent demand

When retailer announces a special sale, consumers' behavior depends on the remaining time of the products lifetime. In this subsection, it is supposed that the average demand per time unit is an increasing function of \((1 - r)\) as follows:

\[
E(x_2) = E(x_1)A(1-r)^B
\]  

where \( A \) and \( B \) are control parameters of the function. By replacing \( E(x_2) \) in Eq. (4) with Eq. (5), the average total revenue function can be stated as follows:

\[
ATR(r, R) = r(v_1 + h)E(x_1) + (1 - r)(v_2 + h)E(x_2) - \left[ (c + h)R + (x + v_2 + h)\beta_1(R) + (x + v_2 + h)\beta_2(R - rE(x_1) + \beta_1(R)) \right]
\]  

The optimal policy for the proposed models are the values of \( r \) and \( R \) maximizing \( ATR(\tau, R) \) functions described in Eqs. (4) and (6), respectively. Behavior of the \( ATR(\tau, R) \) functions depends on the probability distribution of demand while their convexity is not guaranteed, so extracting a closed form optimal solution for a distribution-free structure is impossible.

2.3. Special case 1: uniform distribution

Suppose that product’s demand per time unit before and after sale announcement, \( x_1 \) and \( x_2 \), are uniformly distributed between \((a, b)\) and \((a', b')\), respectively. For the first proposed model, \( a \geq a' \) and \( b \geq b' \) are assumed independent from \((1 - r)\). In this case we have:

\[
ATR(r, R=\frac{(v_1 + b)}{2} - (1 - r)(v_2 + b')\frac{(a' + b')}{2}) - \left[ (c + h)R + (x + v_2 + h)\beta_1(R - \frac{(a' + b')}{2}) + (x + v_2 + h)\beta_2(R - \frac{(a' + b')}{2}) \right]
\]  

For the second proposed model, \( a \geq a' \) and \( b \geq b' \) are assumed
dependent to \((1 - r)\) according to Eq. (5). In this case, the corresponding objective function can be obtained by substituting \(a' = a + 1 - r\) and \(b' = b + 1 - r\) in Eq. (7).

Remark. In the case of uniformly distributed demand, for a given \(r > 0\), \(ATR(R_t)\) is a concave function in terms of \(R\) with a unique optimal value to maximize \(ATR(R_t)\). Therefore, by considering all values of \(r\) between (0,1) the corresponding optimal values of \(R\) is achievable. So, evaluation of the pairs \((r, R)\) in Eq. (7), will result to the optimal values of \((r^*, R^*)\).

2.4. Special case 2: exponential distribution

Let the product's demand per time unit before and after sale announcement, \(x_1\) and \(x_2\), are exponentially distributed with an expected value of \(\mu_1\) and \(\mu_2\), respectively. For the first proposed model, \(\beta_2 \geq \beta_1\) is assumed independent from \((1 - r)\). So we have:

\[
\begin{align*}
ATR(r, R) &= \tau (v_1 + \beta_1) + (1 - r)(v_2 + \beta_2) \\
&= \frac{R}{\tau \beta_1} + \frac{1}{(1 - r)\beta_2} \left( R - \frac{v_1}{\tau \beta_1} + \frac{v_2}{(1 - r)\beta_2} \right) \\
&= \frac{R}{\tau \beta_1} + \frac{1}{(1 - r)\beta_2} \left( R - \frac{v_1}{\tau \beta_1} + \frac{v_2}{(1 - r)\beta_2} \right) \left[ \frac{\tau \beta_1}{R} \right] + \frac{v_2}{(1 - r)\beta_2} \frac{\tau \beta_1}{R} \\
&= \frac{R}{\tau \beta_1} + \left( \frac{R - \frac{v_1}{\tau \beta_1} + \frac{v_2}{(1 - r)\beta_2}}{(1 - r)\beta_2} \right) \left[ \frac{\tau \beta_1}{R} \right]
\end{align*}
\]

For the second proposed model, \(\beta_2 \geq \beta_1\) is assumed dependent to \((1 - r)\) according to Eq. (5). The corresponding objective function can be obtained by substituting \(R = a + 1 - r\) in Eq. (8). Values of parameters \(a\) and \(b\) can be tuned according to the problem instance features.

Remark. In the case of exponentially distributed demand, for a given \(r > 0\), \(ATR(R_t)\) is a concave function in terms of \(R\) with a unique optimal value to maximize \(ATR(R_t)\). Therefore, the same approach as special case 1 can be conducted to obtain the optimal values of \((r^*, R^*)\).

3. Results and discussion

Here the proposed models are validated using numerical examples. In this regard, a solution program is coded using Matlab Software (version R2011a) and an instance for a single-period inventory model is considered with \(c = 10, v_1 = 20, \alpha = 3\) and \(h = 2\). With uniform distribution function for demand in the interval \((a, b)\) = (500,1500), and using Eqs. (1) and (2) the optimal order quantity and the corresponding average total revenue without sale announcement is computed as \(R^* = 1020\) and \(ATR^* = 6880\). Also, applying an exponential distribution with an expected value \(\beta_1 = 1000\), the optimal solution is obtained as \(R^* = 733.97\) and \(ATR^* = 1192.37\).

Now the first proposed model with a sale announcement is applied with uniform and exponential probability distributions for demand. In this regard, assume that for a given sale price \(v_2 = 12\), the parameters of uniform and exponential distributions have been changed to \((a', b')\) = (1000,3000) and \(\beta_2 = 2000\) respectively independent from the sale announcement time. Using Eqs. (7) and (8) the optimal time to announce a special sale, the optimal order quantity and the optimal average total revenue is computed as \(r^* = 0.881\), \(R^* = 1102.9\) and \(ATR^* = 8384.8\) for uniform distribution and \(r^* = 0.808\), \(R^* = 947.1\) and \(ATR^* = 3162.5\) for exponential distribution. Comparison of the results obtained from the first proposed model with the results of the basic single-period model shows that the proposed model with sale announcement leads to greater values of average total revenues. More specifically, using the first proposed model leads to a policy with 21.87% higher \(ATR\) with uniform distribution and 163.23% higher \(ATR\) with exponential distribution than basic model. Concave behavior of \(ATR\) in terms of \(r\) and \(R\) for uniformly distributed demand is depicted in Fig. 1.

In order to evaluate the effect of the problem’s key parameters on the obtained optimal solution, a sensitivity analysis is conducted. In this regard, the first proposed model is solved with different values of three parameters \(c, v_2\) and \((a', b')\) from 50% less to 50% more than their initial values. The results are reported for the uniformly distributed demand in Table 1.

A graphical summary of the results in Table 1 is shown in Fig. 2 demonstrating the trend of \(r^*, R^*\) and \(ATR^*\) when the parameters \(c, v_2\) and \((a', b')\) are altered from −50% to 50%.

It is obvious from Fig. 2 that an increase in unit purchasing price at outset of the period, \(c\), leads to later sale announcement, lower order quantity and reduced average total revenue. Also, Fig. 2 reveals that the more discounted selling price results to earlier sale announcement, greater order quantity and average total revenue. In addition, increased parameters of uniform distribution limits due to the sale announcement culminates in higher \(R^*\), \(r^*\) and \(ATR^*\) with descending intensity.

Application of the first model with different values of three parameters, \(v_2\) and \(\beta_2\) from 50% less to 50% more than their initial values for the exponentially distributed demand leads to the same behavior as uniform distribution which are reported in Table 2.

In continue, the second proposed model with a sale announcement is applied with uniform and exponential probability distributions for demand where average demand depends on product’s remaining lifetime. Assume that for a given sale price \(v_2 = 12\), parameters of uniform and exponential distributions are changed according to the Eq. (5) with tuned control parameters \(A = 400\) and \(B = 3\). Using Eqs. (7) and (8) and substituting \((a', b')\) and \(\beta_2\) according to Eq. (5), the optimal time to announce a special

<table>
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<th>Change %</th>
<th>50%</th>
<th>40%</th>
<th>30%</th>
<th>20%</th>
<th>10%</th>
<th>0%</th>
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<th>20%</th>
<th>30%</th>
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<td>(c)</td>
<td>0.854</td>
<td>0.858</td>
<td>0.863</td>
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<td>0.881</td>
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<tr>
<td>(R) 1200.2</td>
<td>1183.2</td>
<td>1164.8</td>
<td>1145</td>
<td>1124.4</td>
<td>1102.9</td>
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<tr>
<td>(ATR) 1415.7</td>
<td>12962.03</td>
<td>11788.3</td>
<td>10663.8</td>
<td>9498.57</td>
<td>8384.78</td>
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<td>(v_2) 0.932</td>
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<td>0.915</td>
<td>0.905</td>
<td>0.894</td>
<td>0.881</td>
<td>0.867</td>
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<td>0.793</td>
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<tr>
<td>(R) 1020.6</td>
<td>1035</td>
<td>1050.6</td>
<td>1067.2</td>
<td>1084.4</td>
<td>1102.9</td>
<td>1121.6</td>
<td>1140.1</td>
<td>1159.4</td>
<td>1177.9</td>
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<tr>
<td>(ATR) 7373.04</td>
<td>7538.051</td>
<td>7719.91</td>
<td>7920.271</td>
<td>8141.11</td>
<td>8384.78</td>
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<tr>
<td>((a', b')) 0.857</td>
<td>0.863</td>
<td>0.868</td>
<td>0.872</td>
<td>0.877</td>
<td>0.881</td>
<td>0.885</td>
<td>0.889</td>
<td>0.892</td>
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<tr>
<td>(R) 978.5</td>
<td>1008.5</td>
<td>1036.4</td>
<td>1061.5</td>
<td>1083.1</td>
<td>1102.9</td>
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<tr>
<td>(ATR) 7653.56</td>
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<td>8157.23</td>
<td>8278.59</td>
<td>8384.78</td>
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Table 1: Effect of parameters on optimal policy for a uniformly distributed expiry-date-independent demand.
sale, the optimal order quantity and the optimal average total revenue is computed as $r^* = 0.857$, $R^* = 976.7$ and $ATR^* = 7655.1$ for uniform distribution and $r^* = 0.744$, $R^* = 794.1$ and $ATR^* = 2713.4$ for exponential distribution. Comparison of the results obtained from the second model with the results of the basic model shows that the proposed model leads to greater average total revenues for both distributions. More specifically, using the second proposed model leads to a policy with 11.27% and 127.56% higher $ATR$ with uniform and exponential distributions, respectively, which shows lower improvement than the first model. Notice that the optimal order quantity obtained from the both proposed models is more than the basic model for exponential distribution while the optimal order quantity of uniform distribution in the second model is less than the basic model. Concave behavior of $ATR$ for the second model with exponential distribution is depicted in Fig. 3.

The effect of the problem’s key parameters on the obtained optimal solutions of the second model is evaluated. In this regard, the second proposed model with different values of three parameters $c$, $v_2$ and $h$ from 50% less to 50% more than their initial values is solved. The results for the uniformly distributed demand

![Fig. 2. Effect of parameters on optimal policy for a uniformly distributed expiry-date-independent demand.](image-url)
is reported in Table 3.

A graphical summary of the results in Table 3 is shown in Fig. 4 demonstrating the trend of \( \tau^* \), \( R^* \) and \( ATR^* \) when the parameters \( c \), \( v_2 \) and \( h \) are altered from \(-50\%\) to \(50\%\).

Comparison of Fig. 4 with Fig. 2 reveals that there is a same trend in optimal policy as the first model respect to an increase in unit purchasing price at outset of the period, \( c \), and the discounted selling price, \( v_2 \). Here, the unit waste cost of redundant product at end of the period, \( h \), is considered for sensitivity analysis instead of parameters of uniform distribution \( (\alpha, \beta) \). It is clear that an increase in, \( h \), affect slightly on sale announcement time, order quantity and average total revenue in a negative way. Application of the second model with different values of three parameters, \( c \), \( v_2 \) and \( h \) from \(50\%\) less to \(50\%\) more than their initial values for the exponentially distributed demand leads to the same behavior as uniform distribution which are reported in Table 4.

### 4. Summary and conclusion

The effect of sale announcement for perishable items on the average total revenue in a single-period inventory system was investigated in this paper. The importance of sale announcement for perishable items was described and two distribution-free models were presented. Table 2 and Table 3 present the effects of parameters on optimal policy for an exponentially distributed expiry-date-independent demand and a uniformly distributed expiry-date-dependent demand, respectively. Table 2 shows that an increase in \( c \), \( v_2 \) or \( h \) decreases \( \tau^* \), \( R^* \) and \( ATR^* \). Table 3 confirms the same trend but with a different percentage change. The graphical representation in Fig. 4 further illustrates these trends. The results indicate that the optimal policy changes in response to changes in the parameters, as expected. Future research could explore the impact of different distributions and parameter ranges to further validate these findings.
models was proposed to optimize order quantity and sale announcement time. In the first model, average demand after price-cut is assumed independent from time to expiration date while the second model considers a direct relation between them. In order to obtain a closed form solution, two special cases were considered regarding probability distribution of demand after sale announcement. The proposed models were specialized for uniform and exponential distributions and solution approach was presented to optimize them. Application of the proposed models in numerical examples showed that both proposed models result to a higher average total revenue than the basic model. Also, it was seen that the improvement in average total revenue for exponentially distributed demand is considerably more than the uniformly distributed demand. In addition, the optimal order quantity was dependent to demand’s distribution. For exponential distribution both models led to more order quantity than the basic model while this was not true for uniform distribution in the second model. Finally, the influence of the models parameters on

Fig. 4. Effect of parameters on optimal policy for a uniformly distributed expiry-date-dependent demand.
optimal policy was evaluated. It was established that $c$, $v_2$, $\beta_2$ and $(a^*, b^*)$ have notable impact of sale announcement time, order quantity and average total revenue while the influence of $h$ was low. Finding of this paper can help to decision makers in a retail system which deals with perishable products to make an optimal decision to maximize the profitability of their system.

**References**


