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Sakiadis flow of Maxwell fluid considering magnetic field and convective boundary conditions

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In this paper we address the flow of Maxwell fluid due to constantly moving flat radiative surface with convective condition. The flow is under the influence of non-uniform transverse magnetic field. The velocity and temperature distributions have been evaluated numerically by shooting approach. The solution depends on various interesting parameters including local Deborah number De, magnetic field parameter M, Prandtl number Pr and Biot number Bi. We found that variation in velocity with an increase in local Deborah number De is non-monotonic. However temperature is a decreasing function of local Deborah number De. [http://dx.doi.org/10.1063/1.4907927]

I. INTRODUCTION

Non-Newtonian fluid dynamics is one of the most popular research areas of modern fluid mechanics mainly due to its promising applications in chemical and food processing industry. Fluids which change their viscosity or flow behavior under stress are termed as non-Newtonian. A side from air and water, almost all the fluids occurring in industry and biomedicine are non-Newtonian. These include polymers, blood, honey, toothpaste, paints, printer inks, egg whites, engine oil, fruit juices, slurries, shampoos, cosmetic products and many others. Many constitutive relationships of these fluids based on their diverse rheological behaviors exist in the literature. The power-law model is perhaps the most widely discussed non-Newtonian fluid model that has tendency to address the interesting shear-thinning and shear-thickening behaviors. The former is common feature of many non-Newtonian fluids including blood, polymers and paints. However the power-law model is incapable of explaining the visco-elastic effects in the flow. Two visco-elastic fluid models have been consistently used by the researchers namely the (i) second grade model and (ii) the upper-convected Maxwell (UCM) model. While second grade fluid model emphasizes on the normal stress differences, the UCM model can adequately address the characteristics of fluid relaxation time. This model has received wide acceptance in the research community due to its simplicity. Harris[1] formulated the governing equations for two-dimensional flow of Maxwell fluid for the first time. Sadeghy et al.[2] described the classical Sakiadis flow problem involving Maxwell fluid through different analytical and numerical techniques. In another paper, Sadeghy et al.[3] analytically discussed the stagnation-point flow of Maxwell fluid. Kumari and Nath[4] presented an interesting study on mixed convection flow of Maxwell fluid considering magnetic field effects. MHD flow near a stagnation-point towards a porous stretching sheet was considered by Hayat et al.[5] In recent years, various interesting boundary layer flow problems involving Maxwell fluid are addressed (see Refs. 6–17).

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Convective heat transfer has pivotal role in several processes such as transpiration cooling process, material drying, thermal energy storage etc. Therefore it seems reasonable to consider the convective boundary condition instead of isothermal or isoflux wall conditions. Sakiadis and Blasius flow problems with convective boundary conditions have been explored by Bataller.[18] Aziz[19] provided a similarity solution for laminar thermal boundary layer flow over a flat surface using convective boundary condition. He found that temperature across the plate varies with the variation in convective heating parameter. Makinde[20] examined the MHD flow with heat and mass transfer past a moving vertical plate considering convective surface boundary condition. Flow past a convectively heated vertical plate immersed in a porous space was examined by Makinde and Aziz.[21] Magyari[22] derived an analytic solution for Blasius problem with convective boundary condition. Yao et al.[23] presented the exact solutions for flow past a permeable stretching sheet in the presence of convective heating. Recently various interesting problems dealing with the convective boundary condition are addressed (see Refs. 24–29 and several refs. therein).

The purpose here is to examine the Sakiadis flow of electrically conducting Maxwell fluid using convective boundary condition. In accordance with Sadeghy et al.,[2] the coordinate x could not be eliminated from the dimensionless momentum equation. In this situation only local similar solution is possible. Such a solution, if exists, can be used to investigate the influence of parameters at fixed location above the plate. The solutions are successfully computed numerically through shooting approach followed by fourth-fifth-order Runge-Kutta integration method and Newton method. Graphical results are obtained to analyze the underlying physics of the problem.

II. PROBLEM FORMULATION

Consider the two-dimensional flow of Maxwell fluid induced due to a plate moving with a constant velocity U in its own plane as shown in the Fig. 1. The temperature at the plate is passively adjusted through hot convection fluid of temperature T_f. Let T_∞ be the temperature outside the thermal boundary layer. The flow is subjected to transverse magnetic field of strength $B(x) = B_0 x^{-1/2}$. The induced magnetic field is neglected by assuming small magnetic Reynolds number. In the absence of viscous dissipation or heat generation/absorption the boundary layer equations governing the flow and heat transfer in Maxwell fluid can be expressed as (see Shateyi et al.[13]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda_1\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right) = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}\left(u + \lambda_1 v\frac{\partial u}{\partial y}\right), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \tag{3}$$

in which u and v are the velocity components along the x – and y – directions respectively, ν is the kinematic viscosity, λ_1 is the fluid relaxation time, T is the local fluid temperature, α is the thermal diffusivity of the fluid, ρ is the fluid density, c_p is the specific heat and q_r is the radiative heat flux considered as $q_r = -(4\sigma^*/3k^*)\partial T^4/\partial y$,[30] where σ^* and k^* are the Stephan-Boltzmann coefficient and the mean absorption coefficient respectively. Following Raptis and Perdikis,[31] the temperature differences in the flow are assumed to be sufficiently small so that T^4 may be expressed as linear function of temperature. This is accomplished by expanding T^4 about the ambient temperature T_∞ and then neglecting the squares and higher-order terms to obtain $T^4 \cong 4TT_\infty{}^3 - 3T_\infty{}^4$.

The boundary conditions are imposed as below:

$$\begin{gathered} u = U, \quad v = 0, \quad -k\frac{\partial T}{\partial y} = h_f(T_f - T) \quad \text{at } y = 0, \\ u \to 0, \quad T \to T_\infty \quad \text{as } y \to \infty. \end{gathered} \tag{4}$$

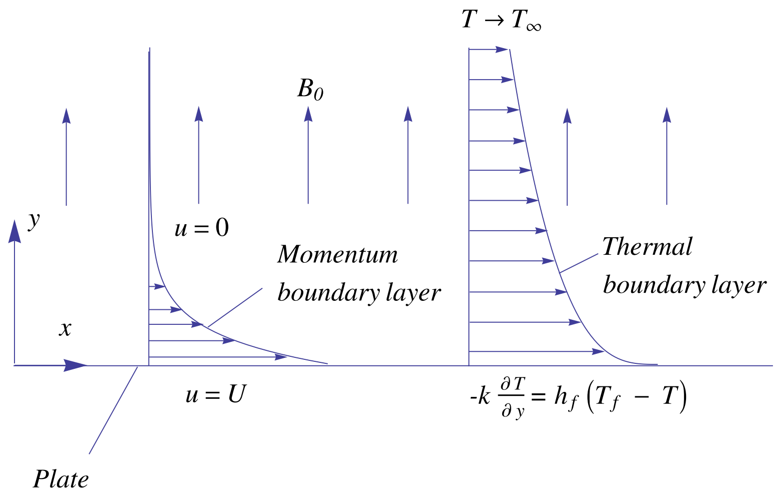

FIG. 1. Physical configuration and coordinate system.

where $h_f = h/\sqrt{x}$ is the heat transfer coefficient. Making use of the following similarity transformations

$$\eta = \sqrt{\frac{U}{\nu x}}y, \quad u = Uf', \quad v = -\frac{1}{2}\sqrt{\frac{\nu U}{x}}\left(f - \eta f'\right), \quad \theta = \frac{T - T_\infty}{T_f - T_\infty}, \tag{5}$$

the continuity Eq. (1) is automatically satisfied and Eqs. (2)–(4) reduce to the following forms

$$f''' + \frac{1}{2}ff'' - \frac{De}{2}\left(2ff'f'' + \eta f'^2 f'' + f^2 f'''\right) - M^2\left(f' - De\left(f - \eta f'\right)f''\right) = 0, \tag{6}$$

$$\left(1 + \frac{4}{3}Rd\right)\theta'' + \frac{Pr}{2}f\theta' = 0, \tag{7}$$

$$\begin{gathered} f(0) = 0, \quad f'(0) = 1, \quad \theta'(0) = -Bi\left(1 - \theta(0)\right), \\ f'(\infty) \to 0 \quad \theta(\infty) \to 0, \end{gathered} \tag{8}$$

where $De = \lambda_1 U/2x$ is the local Deborah number, $M = \sqrt{\sigma B_0^2/\rho U}$ is the magnetic field parameter, $Rd = 4\sigma^* T_\infty^3/kk^*$ is the radiation parameter, $Pr = \nu/\alpha$ is the Prandtl number and $Bi = h/k\sqrt{\nu/U}$ is the Biot number. The quantity of industrial interest is the local Nusselt number Nu_x defined as below

$$Nu_x = \frac{xq_w}{k(T_f - T_\infty)}, \tag{9}$$

where $q_w = -k(\partial T/\partial y)_{y=0} + (q_r)_{y=0}$ is the wall heat flux. Now using Eq. (5), Eq. (9) becomes

$$\mathrm{Re}_x^{-1/2}Nu_x = -\left(1 + \frac{4}{3}Rd\right)\theta'(0). \tag{10}$$

where $\mathrm{Re}_x = Ux/\nu$ is a local Reynolds number.

III. NUMERICAL RESULTS AND DISCUSSION

The numerical solutions of the differential equations (6) and (7) subject to the boundary conditions (8) have been achieved by using shooting approach combined with fourth-fifth-order Runge-Kutta integration and Newton's method. The MATLAB built in routine bvp4c is also used to for obtaining the solutions. We compared our results with the previous study of Cortell[32] in a limiting sense and found an excellent agreement (see Table I). In Table II, we present the numerical values of local Nusselt number $-\theta'(0)$ for different values of embedded parameters. We found that $-\theta'(0)$ has

TABLE I. Computational results of $-\theta'(0)$ for different values of Pr when $M = De = Rd = 0$ and $Bi = 1000000$.

Pr	$-\theta'(0)$ Cortell[32]	$-\theta'(0)$ Present
0.6	0.313519	0.31352
5.5	1.216049	1.21605
7	1.387033	1.38703
10	1.680293	1.68029
50	3.890918	3.89091
100	5.544663	5.54464

TABLE II. Numerical values of $-\theta'(0)$ for different values of De, M, Pr and Rd when $Bi = 0.5$.

De	M	Pr	Rd	*bvp4c*	Shooting Method
0	0	3	0	0.316968	0.316965
			1	0.255219	0.255215
		7	0	0.367517	0.367516
			1	0.316968	0.316965
	1	3	0	0.297051	0.297057
			1	0.222182	0.222240
		7	0	0.357125	0.357125
			1	0.297051	0.297057
1	0	3	0	0.318896	0.318914
			1	0.25415	0.254216
		7	0	0.369191	0.369191
			1	0.318896	0.318914
	1	3	0	0.292616	0.292659
			1	0.212938	0.213447
		7	0	0.355770	0.355773
			1	0.292616	0.292659

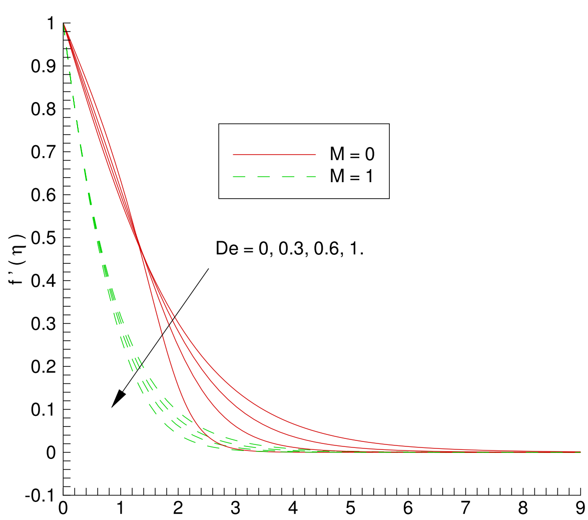

FIG. 2. Effect of De and M on $f'(\eta)$.

direct relationship with local Deborah number De and Prandtl number Pr while it appears to decrease upon increasing either the magnetic field parameter M or the radiation parameter Rd. Interestingly the behavior of De on $-\theta'(0)$ changes as the strength of magnetic field or radiation is gradually increased.

In Fig. 2 the velocity profiles are presented with the variation of the local Deborah number De. Here the two sets of results corresponding to $M = 0$ and $M = 1$ are given. Deborah number is defined

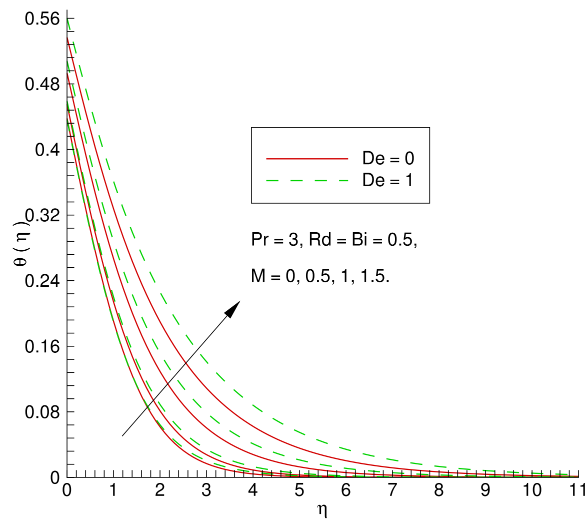

FIG. 3. Effect of De and M on $\theta(\eta)$.

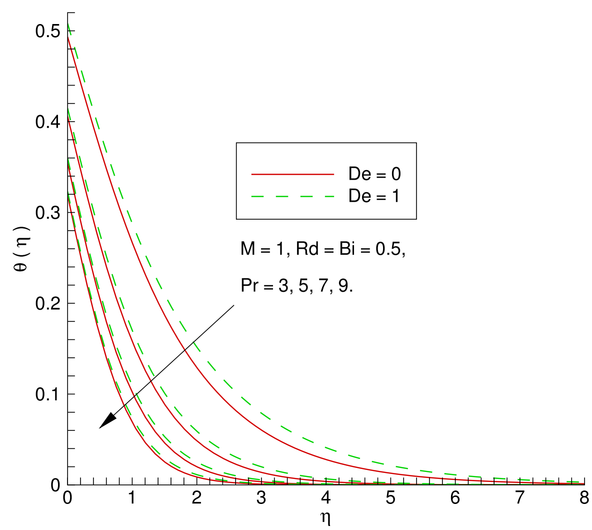

FIG. 4. Effect of Pr on $\theta(\eta)$.

as the fluid relaxation time to the fluid characteristic time and it is small for fluid substances. Interestingly the variation of f' with De is non-monotonic in the absence of magnetic field. The velocity increases with an increase in De just in small region close to the plate whereas it appears to increase with an increase in De in the rest part of the boundary layer. This mixed behavior of De on the velocity can be subdued by strengthening the effect of magnetic field. The decrease in f' is the consequence of larger fluid viscosity associated with larger values of De. We can also observe that profiles descend to zero value at small distances from the plate when either De or M is incremented. This indicates that boundary layer thickness is a decreasing function of both De and M. This outcome is consistent the results of Hsiao[7] in which mixed convection flow of Maxwell fluid is considered.

Fig. 3 contains the influence of magnetic field on the temperature distribution. The resistance associated with the Lorentz force enhances the fluid temperature. Similar pattern of temperature profiles is demonstrated at $De = 0$ and $De = 1$. The temperature drops within the boundary layer when De is increased from $De = 0$ to $De = 1$.

Fig. 4 displays the variation in temperature θ as Prandtl number Pr is incremented. It is quite obvious that Prandtl number has inverse relationship with thermal diffusivity. Due to this reason,

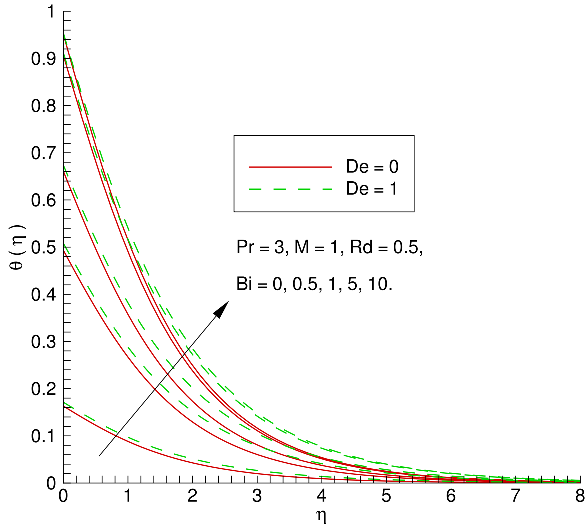

FIG. 5. Effect of Bi on $\theta(\eta)$.

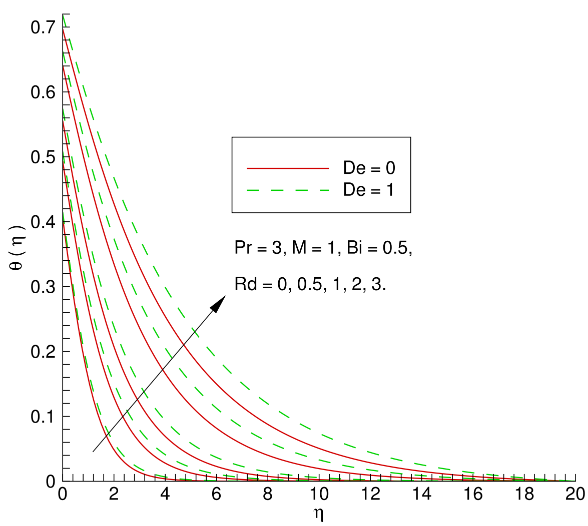

FIG. 6. Effect of Rd on $\theta(\eta)$.

one anticipates a thinner thermal boundary layer in bigger Prandtl number fluid. The reduction in thermal boundary layer results in the large slope of temperature function near the plate. This result is in agreement with Hsiao[10] and Shateyi[13] in which MHD flow of Maxwell fluid is reported.

In Fig. 5, we present the behavior of Biot number Bi on the thermal boundary layer. Biot number is defined as the ratio of convection heat transfer to conduction heat transfer. An increase in Bi implies larger temperature at the plate which creates thicker thermal boundary layer.

It is clear from Fig. 6 that temperature θ appears to increase when radiation parameter Rd is increased. The growth in the thermal boundary layer thickness (with an augmentation in Rd) is accompanied with diminution in the slope of tangent to the curves near the plate.

Fig. 7 presents the local Nusselt number $-\theta'(0)$ as a function of local Deborah number De for different values of Prandtl number Pr and magnetic field parameter M. We notice that $-\theta'(0)$ has linear and direct relationship with De only in the absence of magnetic field. Whereas $-\theta'(0)$ is inversely proportional to De when $M = 1$. In Fig. 8, we found that variation in $-\theta'(0)$ gradually reduces as the radiation parameter Rd is incremented. Fig. 9 gives the influence of convective heating on the heat transfer rate from the plate. We observe that $-\theta'(0)$ increases with an increase in Biot number and

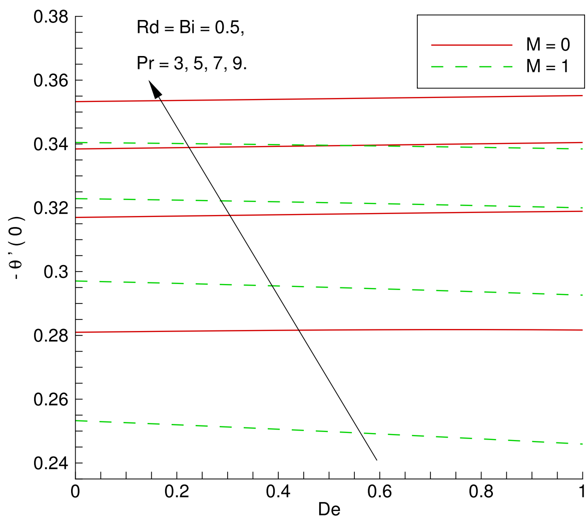

FIG. 7. Effect of Pr and De on $-\theta'(0)$.

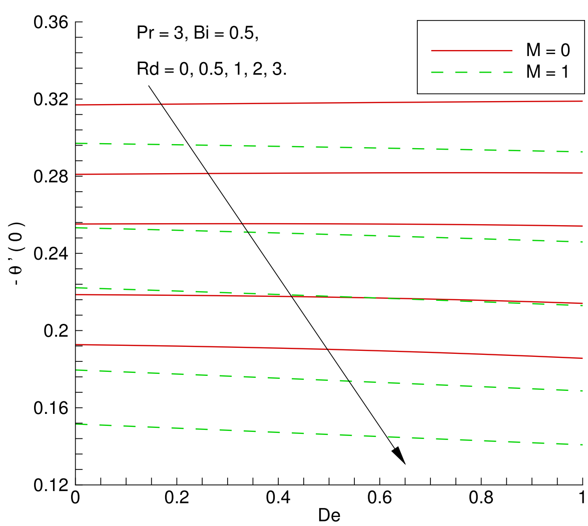

FIG. 8. Effect of Rd and De on $-\theta'(0)$.

approaches to a constant value as $Bi \to \infty$. This constant value is for the constant wall temperature case in which $\theta(0) = 1$.

IV. CONCLUDING REMARKS

MHD flow of Maxwell fluid induced due to constantly moving flat plate is investigated numerically. The novel convective boundary condition is considered in the problem formulation. The inclusion of magnetic field and radiation has significantly influenced the solutions. The main points of this study are summarized as below:

(i) The variation in velocity field f' with an increase in local Deborah number De is non-monotonic when magnetic field effects are absent.
(ii) The thickness of thermal boundary layer is short in Maxwell fluid when compared with the Newtonian fluid.
(iii) Local Nusselt number has inverse relationship with the magnetic field parameter M and the radiation parameter Rd.

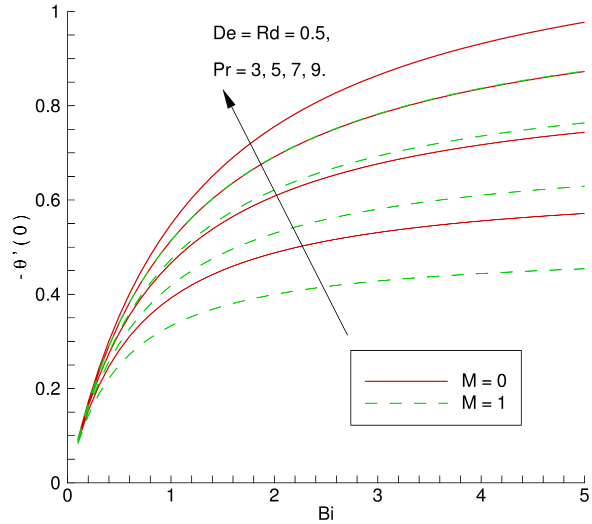

FIG. 9. Effect of De and Bi on $-\theta'(0)$.

(iv) Local Nusselt number is directly/inversely proportional to the local Deborah number De in the absence/presence of magnetic field effects.

(v) The case of constant wall temperature can be studied as special case of present work by assuming sufficiently large Biot number.

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