Transportation Research Part C 31 (2013) 83–98

Contents lists available at SciVerse ScienceDirect

Transportation Research Part C

journal homepage: www.elsevier.com/locate/trc

Speed-based toll design for cordon-based congestion pricing scheme

Zhiyuan Liu [a], Qiang Meng [b,*], Shuaian Wang [c]

[a] Institute of Transport Studies, Department of Civil Engineering, Monash University, Clayton, Vic. 3800, Australia
[b] Department of Civil and Environmental Engineering, National University of Singapore, Singapore 117576, Singapore
[c] School of Mathematics and Applied Statistics, University of Wollongong, Wollongong, NSW 2522, Australia

ARTICLE INFO

Article history:
Received 28 March 2012
Received in revised form 12 December 2012
Accepted 20 February 2013

Keywords:
Cordon-based congestion pricing
Stochastic user equilibrium
Elastic demand
Continuous value-of-time
Distributed genetic algorithm

ABSTRACT

The cordon-based Electronic Road Pricing (ERP) system in Singapore adopts the average travel speed as an index for evaluating the traffic congestion within a cordon area, and the maintenance of the average travel speed within a satisfactory range is taken as the objective of the toll adjustment. To formulate this practical speed-based toll design problem, this paper proposes a mathematical programming with equilibrium constraint (MPEC) model with the objective of maintaining the traffic condition in the cordon area. In the model, the network users' route choice behavior is assumed to follow probit-based stochastic user equilibrium with elastic demand, asymmetric link travel time functions and continuous value-of-time. A distributed revised genetic algorithm is designed for solving the MPEC model. Finally, a network example based on the ERP system is adopted to numerically validate the proposed models and algorithms, and further indicates that the computation speed can be improved greatly by using a distributed computing system.

1. Introduction

The toll design problem for congestion pricing schemes refers to the determination of the optimal toll charge according to one or more objectives, based on given charging locations. Two congestion pricing schemes have received much attention and been comprehensively investigated: first-best (Pigouvian) and second-best pricing (see the monograph by Lewis, 1993; Yang and Huang, 2005; Lawphongpanich et al., 2006; Small and Verhoef, 2007; and a recent review by de Palma and Lindsey, 2011). Some system-wide objectives are usually adopted for the toll design of these two schemes, for instance, the total social benefit, total travel time and toll revenue. However, compared with these system-wide objectives, government and network authorities are usually more concerned about the traffic conditions in a central business district (CBD), the commercial heart of a city, where traffic congestion is likely to cause greater economic losses and worse impacts on a city's image. Thus, regarding the practical implementation of congestion pricing schemes, mitigating traffic congestion in the CBD is usually taken as a primary target.

A cordon-based congestion pricing scheme is advantageous for improving the traffic condition in a CBD as it defines a pricing cordon, encircling a certain area (usually the CBD), and charges each incoming vehicle; the total inbound volume is thus limited and traffic congestion in this area significantly ameliorated. Additionally, area-wide cordon-based pricing schemes are more convenient in terms of operation and supervision, compared with first-best and second-best pricing, which aims to optimize a system-wide objective. Until now, most applications of congestion pricing are cordon-based, for instance the Area Licensing Scheme (ALS) in Singapore (Phang and Toh, 1997; Li, 1999) that was upgraded in 1998 to the

* Corresponding author. Tel.: +65 6516 5494; fax: +65 6779 1635.
E-mail addresses: Zhiyuan.liu@monash.edu (Z. Liu), ceemq@nus.edu.sg (Q. Meng), wangshuaian@gmail.com (S. Wang).

0968-090X/$ - see front matter
http://dx.doi.org/10.1016/j.trc.2013.02.012

Electronic Road Pricing (ERP) system (Olszewski and Xie, 2005), the London Congestion Charging Scheme (Santos, 2008), and a more recent trial in Stockholm (Eliasson, 2009). It should be pointed out that the Stockholm congestion charge scheme also levies tolls on vehicles leaving the charging area.

Average travel speed is an ideal measure of the traffic conditions in an area guarded by a pricing cordon (called a cordon area hereafter), in that it is much easier to observe than traffic column or density (Li, 2002) and is also a better representative of the commuter's driving experience. In the cordon-based ERP system in Singapore, the objective is to keep the average speed of vehicles in the cordon area within a target range: [20, 30] km/h, which is achieved by adjusting the toll charges (Olszewski and Xie, 2005). Note that the lower-bound of this range guarantees a reasonably rapid travel. The upper-bound on travel speed is a concern about traffic safety, and it also avoids a waste of the road resources by ensuring that sufficient vehicles are traveling in the cordon area. Herein, the search for a toll charge pattern that will keep the average travel speed of vehicles in the cordon area within a predetermined target range is named the speed-based toll design problem. Despite its practical significance, the problem is still an open question, since few of the existing studies of toll design problems have taken the traffic conditions in the CBD as an objective or used travel speed as a criterion for network performance.

Modeling the toll design problem requires an analysis of the commuters' route choice problem, and a simple assumption is made here that the commuters will select the path with minimal travel cost based on their pre-trip perceived travel times. The probit-based stochastic user equilibrium (SUE) principle is adopted as a framework for the route choice problem, in view of its better suitability to realistic conditions compared with the deterministic user equilibrium (DUE) and logit-based SUE cases (Sheffi, 1985, p. 318). The commuters' travel costs consist of two components: travel time cost and toll charge, which are expressed in different units. The value-of-time (VOT) is needed to convert the toll charges into time units for the analysis (e.g., Lam and Small, 2001; Yang et al., 2001; Small et al., 2005). The VOT is largely influenced by the commuters' level of income and trip emergency, thus it can vary significantly among commuters. It is difficult to find two commuters on the network with identical VOT values, thus at an aggregate level, it is more suitable to define the VOT as a continuously distributed random variable. The probit-based SUE principle and continuously distributed VOT both increase the challenges involved in modeling and solving the speed-based toll design problem, which are the aims of this paper.

1.1. Relevant studies

It has been well recognized that marginal cost pricing is a solution to the first-best pricing scheme with the objective of optimizing a system-wide index such as the total social benefit or total travel time (Yang and Huang, 2005; Lawphongpanich and Yin, 2012). The validity of marginal cost pricing for general transportation networks has been proven by many studies under different assumptions, for example, with elastic demand (Huang and Yang, 1996), with logit-based SUE constraints (Yang and Huang, 1998), with general SUE constraints (Maher et al., 2005), and with stochastic demand (Sumalee and Xu, 2011), to name a few. The optimal marginal cost pricing scheme can easily be obtained by solving a traffic assignment problem. An engineering-oriented trial-and-error method was proposed by Yang et al. (2004) and Zhao and Kockelman (2006) with DUE constraints and logit-based SUE constraints respectively, where the travel demand is not required for the calculation. The work of Yang et al. (2004) was recently extended by Han and Yang (2009) and Yang et al. (2009) using more efficient step sizes.

Marginal cost pricing requires each link to be charged, thus it is not practical in real life. If one assumes that only a proportion of the network is charged, the second-best pricing scheme can be obtained (Yang et al., 2010). Second-best pricing problems can be formulated as a bi-level programming model, where the upper-level is to optimize any given system-wide index and the lower-level is a traffic assignment problem. The lower-level problem can be treated as a constraint on the upper-level problem, giving a form of mathematical programming with equilibrium constraints (MPEC) model (see, e.g., McDonald, 1995; Bellei et al., 2002; Chen and Bernstein, 2004, to name a few). The bi-level programming or MPEC model can be solved by various methods, including the iterative optimization-assignment algorithm (Allsop, 1974), equilibrium decomposed optimization (Suwansirikul et al., 1987), the sensitivity-analysis-based algorithm (Yang, 1997; Clark and Watling, 2002; Connors et al., 2007), the augmented Lagrangian algorithm (Meng et al., 2001) and gradient-based descent methods (Chiou, 2005). Although the cordon-based pricing scheme is a special type of second-best pricing, the aforementioned methods cannot be used for the speed-based toll design problem addressed in this study, due to the existence of continuously distributed VOT.

As mentioned above, the VOT is used to convert the toll charges into time units so as to analyze the commuters' route choice problem. The VOT is inherently influenced by many factors, including wage rate, time of day, trip purpose, importance of travel time reliability, etc. Thus, VOT can vary widely between commuters. It is thus more rational to take VOT to be a continuously distributed random variable across the population instead of assuming homogeneous network users with constant VOT or limited user classes with discrete VOTs (Han and Yang, 2008). Yet, studies of congestion pricing problems, or any other transportation network modeling problems, with continuously distributed VOT are quite scarce. Mayet and Hansen (2000) analyzed the toll design problem with continuous VOT on a network with two paths: one congested highway with a toll charge and one alternative path with a fixed travel cost. Expressions for the toll charge that maximizes the total user benefit were given by Mayet and Hansen, and they also discussed the effects of the distribution of VOT on the Pareto properties of the toll charge. Also based on the two-path example, Verhoef and Small (2004) investigated second-best pricing with a continuously distributed VOT. Xiao and Yang (2008) extended the work of Mayet and Hansen (2000) to cope with build-operate-transfer (BOT) contracts for highway franchising programs with continuously distributed VOT. Nie and Liu

(2010) recently conducted a more in-depth analysis about the impacts of various distributions of VOT on the Pareto-improving congestion pricing scheme. However, these studies all rely on a network with two paths, while for a general transportation network with more than two paths, the findings may not apply. For a general transportation network, assuming the DUE principle, Leurent (1993) and Dial (1996, 1997) discussed the traffic assignment problem with continuously distributed VOT. Assuming probit-based SUE with fixed demand, Cantarella and Binetti (1998) later investigated a mathematical model and solution algorithm for the traffic assignment problem with continuously distributed VOT, using a path-based Monte Carlo simulation method to solve the stochastic network loading (SNL) problem. Meng et al. (2012) extended the work of Cantarella and Binetti (1998) by proposing a link-based Monte Carlo simulation method, which is modified and employed in this paper to solve the speed-based toll design problem with continuous VOT and SUE constraints.

1.2. Objectives and contributions

This paper works on the model design and algorithm development for the speed-based toll design problem. A toll charge pattern that keeps the average travel speed in the cordon area within a predetermined target range is regarded as "acceptable". However, for any given transportation network, there is likely to be more than one acceptable toll charge pattern. Thus, the optimum among these acceptable patterns that gives rise to the highest total social benefit (TSB) is regarded as the solution to the speed-based toll design problem. A MPEC model is first proposed for the problem, wherein the equilibrium constraints represent a probit-based SUE problem with continuously distributed VOT. Elastic demand and asymmetric link travel time functions are further assumed for the SUE problem that is then formulated as a fixed-point model with unique solution. A convergent cost averaging (CA) method (Cantarella, 1997), incorporating a two-stage Monte Carlo simulation-based stochastic network loading method, is then adopted to solve the SUE problem. Due to the existence of continuously distributed VOT, existing algorithms are not available for solving the proposed MPEC model. Thus, a genetic algorithm type of method is taken as a heuristic for solving the speed-based toll design problem.

It should be highlighted that an engineering-oriented approach is currently used to adjust the toll charges in the ERP system in Singapore (Olszewski and Xie, 2005): a review is conducted every 3 months of the average travel speed in each cordon area, and then toll charges on all entry points to each cordon are adjusted accordingly, that is, increased (reduced) by a certain amount if the average travel speed is less than the lower bound (greater than the upper bound) of the target range. This trial-and-error type of toll adjustment approach is quite convenient in practice, and is thus incorporated in the solution algorithm used in this paper, which is named the revised genetic algorithm.

Most of the computational effort in the revised genetic algorithm is spent on the evaluation of each newly generated chromosome, which is a SUE problem with given toll charges. Due to the high computational cost of the Monte Carlo simulation in each iteration of the CA method, it is computationally prohibitive to perform the revised genetic algorithm on middle- or large-scale transportation networks. Yet, due to the complete independence of each evaluation process, the revised genetic algorithm can be considerably accelerated by means of distributed computing. The performance and improved computation speed of this distributed revised genetic algorithm (DRGA) is sensitive to the number of processors used, and should be further tested in numerical experiments.

The remaining sections are organized as follows. Section 2 presents the specific notations and assumptions, and then proposes a MPEC model for the speed-based toll design problem. Section 3 addresses the probit-based SUE problem with continuously distributed VOT, elastic demand and asymmetric link travel time functions, where a fixed-point model and corresponding solution algorithm are given. Section 4 discusses the use of the DRGA for solving the speed-based toll design problem, and this is numerically tested using a network example in Section 5. The paper is concluded in Section 6.

2. Problem statement and MPEC model for speed-based toll design

2.1. Notation and definitions

Let $G = (N, A)$ denote a strongly connected transportation network, where N and A are the sets of nodes and directed links, respectively. For the cordon-based congestion pricing scheme, toll charges are implemented at each entry of the pricing cordon. Let $\overline{A}$ be the set of all charging links, thus $\overline{A} \subseteq A$. The toll fare on each link $a \in \overline{A}$ is denoted by τ_a, and for the sake of presentation, all the toll fares are grouped into a column vector $\boldsymbol{\tau} = (\tau_a, a \in \overline{A})^{\mathrm{T}}$.

Assume that the total number of pricing cordons is I, and each cordon is sequentially numbered with an integer from 1 to I. Any toll fare pattern is regarded as "acceptable" if the average travel speed of the vehicles in each pricing cordon remains within a target range. That is, for the ith pricing cordon (called cordon i hereafter), $1 \leqslant i \leqslant I$, if the average speed of all the vehicles in this cordon is denoted by γ_i, we have that $\underline{\gamma}_i \le \gamma_i \le \bar{\gamma}_i$, where constant $\underline{\gamma}_i$ ($\bar{\gamma}_i$) is a predetermined lower (upper) bound of the target range for average speed γ_i.

γ_i will be inherently influenced by the toll charges $\boldsymbol{\tau} = (\tau_a, a \in \overline{A})^{\mathrm{T}}$, which affect commuters' route choices and eventually change the flows and travel speeds on each link in the cordon area. Any variation in the toll fares could lead the commuters to adjust their route choices, and the link flows are assumed to converge to a new equilibrium after a short span of time. Thus, let $v_a(\boldsymbol{\tau})$ denote the equilibrium flow on link $a \in A$, which should be a function of the toll charge pattern $\boldsymbol{\tau}$. All the link flows are grouped into a column vector $\mathbf{v}(\boldsymbol{\tau}) = (v_a(\boldsymbol{\tau}), a \in A)^{\mathrm{T}}$. Likewise, we can define the following attributes on network $G = (N, A)$:

W	Set of Origin-Destination (OD) pairs
$q_w(\tau)$	Travel demand between OD pair $w \in W$
$\mathbf{q}(\tau)$	Column vector of all the OD travel demands, $\mathbf{q}(\tau) = (q_w(\tau), w \in W)^T$
R_w	Set of all the paths between OD pair $w \in W$
$f_{wk}(\tau)$	Traffic flow on path $k \in R_w$ between OD pair $w \in W$
$\mathbf{f}(\tau)$	Column vector of traffic flows on all the paths, $\mathbf{f}(\tau) = (f_{wk}(\tau), k \in R_w, w \in W)^T$
$t_a(\mathbf{v})$	Travel time on link $a \in A$, and it is a strictly increasing and continuously differentiable function of link flow vector $\mathbf{v}$
$\mathbf{t}(\mathbf{v})$	Column vector of all the link travel time functions, $\mathbf{t}(\mathbf{v}) = (t_a(\mathbf{v}), a \in A)^T$
$c_{wk}(\mathbf{v})$	Travel time on path $k \in R_w$, and travel times on all the paths between OD pair $w \in W$ are grouped into vector $\mathbf{c}_w(\mathbf{v}) = (c_{wk}(\mathbf{v}), k \in R_w)^T$.

It should be noted that the link travel time vector $\mathbf{t}(\mathbf{v})$ is allowed to have either a symmetric or asymmetric Jacobian matrix *w.r.t.* the link flow vector $\mathbf{v}$, which is usually referred to as asymmetric link travel time functions in the literature. In light of the flow conservation conditions, the following equations should be satisfied:

$$\mathbf{q}(\tau) = \Lambda \mathbf{f}(\tau) \tag{1}$$

$$\mathbf{v}(\tau) = \Delta \mathbf{f}(\tau) \tag{2}$$

$$\mathbf{f}(\tau) \geq 0 \tag{3}$$

Here, $\Lambda = [\delta_k^w]_{|W| \times K}$ and $\Delta = [\delta_{ak}^w]_{|A| \times K}$ are the incidence OD-pair/path and link/path matrices, where $|A|$, $|W|$ and K are the cardinality of the set of links, the OD pairs and the paths, respectively. $\delta_k^w = 1$ if path k connects OD pair $w \in W$, and $\delta_k^w = 0$, otherwise. Meanwhile, $\delta_{ka}^w = 1$ if link a is on path $k \in R_w$, and $\delta_{ka}^w = 0$ otherwise.

The path travel time $c_{wk}(\mathbf{v})$ is defined as the summation of the travel times on all the links on this path, thus,

$$c_{wk}(\mathbf{v}) = \sum_{a \in A} t_a(\mathbf{v}) \delta_{ak}^w, k \in R_w, \; w \in W \tag{4}$$

The cumulative toll charges, denoted by τ_{wk}, on path $k \in R_w$ can be calculated by:

$$\tau_{wk} = \sum_{a \in \bar{A}} \tau_a \delta_{ak}^w \tag{5}$$

To analyze the impacts of these toll fares on commuters' route choices, τ_{wk} should be converted into time units using the commuters' VOT. As discussed in Section 1.1, the VOT, denoted by α, is regarded as a continuously distributed random variable across the whole population for all the OD pairs. We further assume that α has flow-independent mean and variance, and its probability density function (PDF) is continuously differentiable.

The commuters make their per-trip route plans based on their perceived value of the costs on each path, and this perceived cost on path $k \in R_w$ is:

$$C_{wk}(\mathbf{v}, \tau) = c_{wk}(\mathbf{v}) + \zeta_{wk} + \frac{\tau_{wk}}{\alpha} \tag{6}$$

where ζ_{wk} denotes the commuters' perception error regarding the path travel time, which is a random variable with zero mean and flow-independent variance. Compared with the standard SUE problem, the path travel cost here has another random term, the VOT. Thus, the network equilibrium in this case is named the generalized SUE, which was initially proposed by Meng et al. (2012) to analyze the distance-based toll design problem. In this paper, the perception error is assumed to be normally distributed, which gives the generalized probit-based SUE. Meanwhile, the models and algorithms proposed are all suitable for DUE or logit-based SUE cases.

Let $S_w(\mathbf{v}, \tau)$ be the mean value of the minimal generalized path travel time between OD pair $w \in W$, which is usually called the satisfaction function (Sheffi, 1985), namely,

$$S_w(\mathbf{v}, \tau) = E\left[\min_{k \in R_w} \{C_{wk}(\mathbf{v}, \tau)\}\right] \tag{7}$$

$S_w(\mathbf{v}, \tau)$ can be used to measure the impedance of traveling between OD pair $w \in W$. Thus, travel demand between any OD pair $w \in W$ is rationally assumed to be a function of $S_w(\mathbf{v}, \tau)$, with the expression:

$$q_w = D_w(S_w(\mathbf{v}, \tau)) \leq \bar{q}_w, \; w \in W \tag{8}$$

where the demand function $D_w(\bullet)$ is assumed to be continuously differentiable and non-increasing (Cantarella, 1997; Maher and Zhang, 2000). $\bar{q}_w$ is a predetermined upper bound on the travel demand, which is dependent on the population and car-ownership in the origin zone.

2.2. MPEC model for the speed-based toll design problem

With a predetermined target range for the average speed of vehicles in each pricing cordon, $\underline{\gamma}_i \le \gamma_i \le \bar{\gamma}_i$, there will be more than one "acceptable" toll fare pattern that generates a desirable average travel speed in all cordons. Thus, the toll design problem, as explained in Section 1.2, searches for the "optimal" toll fare pattern among all the acceptable solutions that gives rise to the maximum TSB. In the context of a generalized probit-based SUE problem with elastic demand and asymmetric link travel time function, an expression of TSB can be given as follows (Meng et al. 2012):

$$Z_1(\boldsymbol{\tau}) = \sum_{w \in W} \int_0^{q_w(\boldsymbol{\tau})} D_w^{-1}(x)dx - \sum_{w \in W} q_w(\boldsymbol{\tau}) S_w(\mathbf{v}, \boldsymbol{\tau}) + \sum_{a \in \bar{A}} E\Big(v_a(\boldsymbol{\tau}) \frac{\tau_a}{\alpha} \Big) \quad (9)$$

where the first term on the right-hand-side of Eq. (9) is the overall benefits obtained by the commuters from their trips, and the second term represents the overall travel costs borne by the commuters. The last term is the total toll revenue.

Based on Eq. (9), the speed-based toll design problem can then be formulated by the following MPEC model:

$$\max_{\boldsymbol{\tau} \in \Omega} Z_1(\boldsymbol{\tau}) \quad (10)$$

subject to

$$\underline{\gamma}_i \le \gamma_i(\boldsymbol{\tau}) \le \bar{\gamma}_i, i = 1, 2, \ldots, I \quad (11)$$

$$\mathbf{v}(\boldsymbol{\tau}), \mathbf{f}(\boldsymbol{\tau}) \text{ and } \mathbf{q}(\boldsymbol{\tau}) \text{ fulfill the stochastic user equilibrium conditions} \quad (12)$$

In model (10), Ω denotes the feasible set of toll fare patterns, that is, $\Omega = \{\boldsymbol{\tau} | \underline{\tau} \le \tau_a \le \bar{\tau}, a \in \bar{A}\}$, where $\underline{\tau}$ ($\bar{\tau}$) is a predetermined lower (upper) bound on the toll charges. In (12), $\mathbf{v}(\boldsymbol{\tau})$, $\mathbf{f}(\boldsymbol{\tau})$ and $\mathbf{q}(\boldsymbol{\tau})$ denote the vectors of link flows, path flows and OD demands, which can be attained by solving a generalized probit-based SUE problem. The mathematical model and solution algorithm for the generalized probit-based SUE problem are thus investigated in the following section.

3. Generalized probit-based stochastic user equilibrium

3.1. Fixed-point model

Equilibrium flows on a network with random path travel times were analyzed by Daganzo and Sheffi (1977) by means of the discrete choice model, giving the general SUE principle. In terms of the generalized path travel time function defined in Eq. (6), the equilibrium flows of the generalized probit-based SUE problem can also be formulated using the discrete choice model. Namely, with flow-independent values of generalized path travel time $\mathbf{C}_w = (C_{wk}, k \in R_w)^{\mathrm{T}}$, let p_{wk} denote the probability that path $k \in R_w$ is perceived as the one with the minimal cost among all the paths between OD pair w:

$$p_{wk} = \Pr(C_{wk} \leqslant C_{wj}, \forall j \in R_w \text{ and } j \ne k), k \in R_w, \ w \in W \quad (13)$$

Herein, p_{wk} is called the path choice probability. Then, the corresponding flow on path $k \in R_w$ is defined as

$$f_{wk} = D_w(S_w) \times p_{wk}, k \in R_w, \ w \in W \quad (14)$$

In accordance with the flow conservation equations, we can see that the flow on link $a \in A$ equals

$$v_a = \sum_{w \in W} D_w(S_w) \times \sum_{k \in R_w} p_{wk} \delta_{ak}^w \quad (15)$$

A feasible set of link flows, denoted by $\Omega_{\mathbf{v}}$, can be defined as:

$$\Omega_{\mathbf{v}} = \left\{ \mathbf{v} \middle| v_a = \sum_{w \in W} \sum_{k \in R_w} f_{wk} \delta_{ak}^w, a \in A, \sum_{k \in R_w} f_{wk} = q_w, q_w \in [0, \bar{q}_w], w \in W, \ f_{wk} \geqslant 0 \right\} \quad (16)$$

The flow pattern $\mathbf{v} = (v_a, a \in A)^{\mathrm{T}}$ yielded by Eq. (15) will, in turn, affect the travel times on the network. Moreover, values for p_{wk} and S_w should also be updated iteratively. A fixed-point model can be defined on set $\Omega_{\mathbf{v}}$ to address the equilibrium link flow for any given toll charge pattern $\boldsymbol{\tau} = (\tau_a, a \in \bar{A})^{\mathrm{T}}$: any link flow pattern $\mathbf{v} \in \Omega_{\mathbf{v}}$ is a solution of the generalized probit-based SUE problem if it satisfies the following equation:

$$v_a(\boldsymbol{\tau}) = \sum_{w \in W} \sum_{k \in R_w} [D_w(S_w(\mathbf{v}, \boldsymbol{\tau})) \times p_{wk}(\mathbf{v}, \boldsymbol{\tau}) \delta_{ak}^w], \ a \in A \quad (17)$$

As per Theorems 1 and 2 of Cantarella (1997), this fixed-point model has a unique solution if the demand functions are non-increasing and the link travel time functions strictly increasing. The monotonicity of the demand functions and the link travel time functions can be guaranteed based on the assumptions made in Section 2. The CA method proposed by Cantarella (1997) can then be used to solve the fixed-point model defined in Eq. (17). Due to space limits, the detailed procedures of the CA method are not included here.

The CA method iteratively invokes a stochastic network loading method. Stochastic network loading is, by nature, the result of the discrete choice model based on fixed generalized path travel times, as presented in Eq. (14). The solution method for the stochastic network loading of the generalized probit-based SUE problem with elastic demand and asymmetric link travel time functions is discussed in the following sub-section.

3.2. Two-stage monte carlo simulation based stochastic network loading method

In previous studies, SNL of probit-based SUE problem is usually solved by two types of methods: Monte Carlo simulation based method (Sheffi and Powell, 1981; Meng and Liu, 2011) or some approximation methods (Maher and Hughes, 1997; Rosa and Maher, 2002). However, for the generalized probit-based SUE problem, the existing approximation methods are invalid, due to the randomly distributed VOT. Thus, in this study, we decide to use Monte Carlo simulation for solving the SNL of generalized probit-based SUE problem. Yet, since the perception errors on travel time ζ_{wk}, see Eq. (6), are defined on paths rather than links, directly using Monte Carlo simulation would require the path enumeration or path generation. To cope with this, a link-based interpretation of ζ_{wk} (see Section 11.2 of Sheffi, 1985; Liu and Meng, 2012) is used to convert it into link-based values, which enables a link-based Monte Carlo method for solving the SNL.

Shown as follows, we assume that the commuters' perceived travel time on link $a \in A$ equals to:

$$T_a(\mathbf{v}) = t_a(\mathbf{v}) + \xi_a,\ a \in A \tag{18}$$

where the perception error ξ_a is normally distributed with zero mean and constant variance, that is:

$$\xi_a \sim N(0, \beta t_a^0),\ a \in A \tag{19}$$

where parameter β is a constant and t_a^0 is the free-flow link travel time.

Based on Eq. (18), we can design a link-based two-stage Monte Carlo simulation method for solving the SNL of generalized probit-based SUE problem, based on given link travel times t_a, $a \in A$. The first stage is used to estimate the value of OD demand. Then, with this OD demand, the second stage aims to estimate the corresponding link flows. Procedures of this simulation are summarized as follows:

3.2.1. Stage 1: Monte-Carlo simulation for calculating travel demand

Step 1.0: (Initialization). Let the index of simulation $n = 1$ and the initial estimated satisfaction $\overline{S}_w^{(0)} = 0, w \in W$.
Step 1.1: (Sampling of link travel time). For each link $a \in A$, sample a link travel time denoted by $\tilde{t}_a^{(n)}$ from the normally distributed population $N(t_a, \beta t_a^0)$.
Step 1.2: (Travel cost of dummy links) Sample a value for VOT $\alpha^{(n)}$ based on its distribution function and calculate the generalized link travel cost:

$$\tilde{T}_a^{(n)} = \begin{cases} \tilde{t}_a^{(n)} + \frac{\tau_a}{\alpha^{(n)}}, a \in \overline{A} \\ \tilde{t}_a^{(n)}, a \in A \setminus \overline{A} \end{cases}, \quad a \in A \tag{20}$$

Step 1.3: (Shortest path time calculation). With generalized link travel cost pattern $\{\tilde{T}_a^{(n)}, a \in A\}$, calculate travel cost of the shortest path between each OD pair $w \in W$, denoted by $\tilde{C}_w^{(n)}$, namely,

$$\tilde{C}_w^{(n)} = \min_{k \in R_w} \left(\tilde{c}_{wk}^{(n)} = \sum_{a \in A} \tilde{T}_a^{(n)} \delta_{ak}^w \right),\ w \in W \tag{21}$$

Step 1.4: (Satisfaction estimation). Estimate the satisfaction for each OD pair $w \in W$ by the average scheme:

$$\overline{S}_w^{(n)} = \frac{(n-1)\overline{S}_w^{(n-1)} + \tilde{C}_w^{(n)}}{n}, \quad w \in W \tag{22}$$

Step 1.5: (Accuracy checking). If the number of iterations $n \geqslant n_{\max}$, where $n_{\max}$ is a predetermined sample size, go to Step 1.6; otherwise, set $n = n + 1$ and go to Step 1.1.
Step 1.6: (OD demand calculation). Calculate OD travel demand pattern by the formulae:

$$\tilde{q}_w = D_w(\overline{S}_w^{(n)}),\ w \in W \tag{23}$$

Then go to Stage 2, and use this fixed travel demand value $\tilde{q}_w,\ w \in W$ to simulate the link flow.

3.2.2. Stage 2: Monte-Carlo simulation for the link flows

Step 2.0: (Initialization). Set the initial link travel flow vector $v_a^{(0)} = 0,\ a \in A$ and the index of simulation $l = 1$.

Step 2.1: (Sampling of link travel time). Sample the link travel times $\tilde{t}_a^{(l)},\ a \in A$ from $N(t_a, \beta t_a^0),\ a \in A$ based on normally distributed random number series.
Step 2.2: (Sampling of VOT). Sample value for VOT $\alpha^{(l)}$ from its distribution function and calculate the generalized link travel time:

$$\tilde{T}_a^{(l)} = \begin{cases} \tilde{t}_a^{(l)} + \frac{\tau_a}{\alpha^{(l)}}, a \in \overline{A} \\ \tilde{t}_a^{(l)}, a \in A \setminus \overline{A} \end{cases}, \quad a \in A \tag{24}$$

Step 2.3: (All-or-nothing traffic assignment). (i) Define an initial OD pair based link flow solution:

$$y_{aw}^{(l)} = 0, a \in A, w \in W \tag{25}$$

(ii) With generalized link travel time pattern $\{\tilde{T}_a^{(l)}, a \in A\}$, find the shortest path for each OD pair w, then assign OD travel demand $\tilde{q}_w$ calculated in Step 1.6 to each link of the shortest path, namely,

$$y_{aw}^{(l)} = \tilde{q}_w, \text{for any link} a \text{on the shortest path between OD pair} w \in W \tag{26}$$

(iii) Summing up traffic flow of each link yields the auxiliary link flow pattern $\{y_a^{(l)} = \sum_{w\in W} y_{aw}^{(l)},\ a \in A\}$.
Step 2.4: (Link flow estimation). Calculate the stochastic network loading flows by the averaging scheme:

$$\bar{v}_a^{(l)} = \frac{(l-1)\bar{v}_a^{(l-1)} + y_a^{(l)}}{l}, \quad a \in A \tag{27}$$

Step 2.5: (Stop criterion check). If the number of iterations $l \geqslant l_{\max}$, where $l_{\max}$ is the predetermined sample size, then stop and output the link flow values $\{v_a(\boldsymbol{\tau}) = \bar{v}_a^{(l)},\ a \in A\}$. Otherwise, let $l = l + 1$ and go to Step 2.1.

4. Solution algorithm for the MPEC model

In view that the proposed MPEC model, Eqs. (10)–(12), is not convex and also considering the continuously distributed VOT, existing algorithms (see Section 1.1) are not available for solving this MPEC model. Note that a theoretically effectual method is to enumerate all the feasible toll patterns, assess their corresponding value of total social benefit (TSB) and average speed $\gamma_i(\boldsymbol{\tau})$, and then choose the optimum with maximal TSB value, among those toll patterns that can fulfill the desired speed interval, Eq. (11). This brute force method is extremely time-consuming, per se, and would be computationally prohibitive even for a medium size example. Consequently, in this paper we adopt a genetic algorithm (GA) type method to solve the proposed model, which is an approximation of the brute force method.

GA is one of the most well-known search heuristics for solving optimization problems (e.g., Goldberg, 1989; Gen and Cheng, 1997). Chromosomes of the GA are designed in this way: all the tolled links on the network are successively numbered, and each gene in one chromosome represents the toll charge on the corresponding tolled link. For the chromosomes in the initial generation, all their genes are randomly generated between $\underline{\tau}$ and $\bar{\tau}$.

To cope with the speed constraint (11), a penalty cost is added to the objective function, thus the model (10) is approximated by the following model:

$$\max_{\boldsymbol{\tau}\in\Omega} Z_2(\boldsymbol{\tau}) = Z_1(\boldsymbol{\tau}) - c\sum_{i=1}^{I} \max(0, \underline{\gamma}_i - \gamma_i, \gamma_i - \bar{\gamma}_i) \tag{28}$$

subject to:

$$v_a(\boldsymbol{\tau}) = \sum_{w\in W}\sum_{k\in R_w} [D_w(S_w(\mathbf{v}, \boldsymbol{\tau})) \times p_{wk}(\mathbf{v}, \boldsymbol{\tau})\delta_{ak}^w], \quad a \in A \tag{29}$$

where penalty parameter c is a large positive number.

4.1. Revised genetic algorithm

As shown in Section 1.2, the Land Transport Authority in Singapore adjusts the toll charges based on a regular survey on the travel speed. In accordance with this strategy for toll adjustment, for any chromosome in a particular generation of the GA, if its corresponding average speed is not in the targeted range, a similar adjustment would be conducted on the chromosomes. This toll adjustment strategy would subsequently produce a new chromosome. Together with those from the crossover and mutation processes, all the newly generated chromosomes will be considered for the selection of next generation. Such a solution algorithm for the speed-based toll design problem is called as Revised Genetic Algorithm, shown as follows:

Step 1: (Initial population). Set the size of population to be $\bar{n}$. Randomly generate initial population of the chromosomes, which contains toll fares on each tolled link. Let number of generation $\bar{m} = 1$.
Step 2: (Crossover). Randomly choose some parents from the survivors, and conduct pairing between each parent, which yields some new chromosomes.
Step 3: (Mutation). With a lower probability, randomly choose some genes from all the chromosomes in current generation, and then modify value of these genes by a pseudo random number between $\underline{\tau}$ and $\bar{\tau}$. This process also generates some new chromosomes.
Step 4: (Evaluation). For each newly generated chromosome, perform a generalized probit-based SUE traffic assignment using the CA method, and then record its corresponding TSB value in terms of the objective function (28).
Step 5: (Toll adjustment). Based on the existing individuals in current generation, perform a one-off adjustment on their toll fares in turns: for a chromosome with the toll fares equal to $\boldsymbol{\tau} = (\tau_a,\ a \in \overline{A})^{\mathrm{T}}$, check the corresponding average speed of vehicles in pricing cordon i, denoted by $\gamma_i(\boldsymbol{\tau})$, and if $\gamma_i(\boldsymbol{\tau})$ is less than the predetermined lower bound $\underline{\gamma}_i$, then increase toll fares on all the entry links to this cordon by π; otherwise if $\gamma_i(\boldsymbol{\tau})$ is greater than its upper bound $\bar{\gamma}_i$, deduct the toll fares on its entry links by π. Here, the increment π is predetermined and fixed. This adjustment produces some new chromosomes, and we then evaluate the TSB value of these new chromosomes.
Step 6: (Selection). Among all the existing individuals, choose the top $\bar{n}$ individuals with larger TSB values, and then set these $\bar{n}$ individuals as survivors for next generation.
Step 7: (Stop Test). If $\bar{m} > \bar{m}_{\max}$, then stop, where $\bar{m}_{\max}$ is a predetermined upper-bound for the number of generations; otherwise, set $\bar{m} = \bar{m} + 1$ and go to Step 2.

4.2. Decomposition of revised genetic algorithm for distributed computing

It can be seen from the procedures above for Revised Genetic Algorithm that evaluation process of each chromosome mainly requires solving a generalized probit-based SUE traffic assignment based on given toll pattern $\boldsymbol{\tau} \in \Omega$. This SUE problem is solved by the CA method proposed by Cantarella (1997). However, since Monte Carlo simulation is adopted for the stochastic network loading of CA method, computational cost of the Revised Genetic Algorithm would be tremendously large even for a medium size network. In fact, this hurdle of prohibitive computational time occurs in many applications of GA on transportation networks.

Despite of this seeming difficulty, we can see that evaluation for each chromosome is independent, and the computation tasks are identical for the evaluation of each newly generated chromosome. It is thus quite straightforward to conduct the calculation simultaneously by various processors in a distributed computing system, and such a computation procedure for solving the speed-based toll design problem is named as Distributed Revised Genetic Algorithm (DRGA). Regarding the parallel (distributed) GA type methods used in transportation studies, Wong et al. (2001) proposed a parallel GA for the calibration of Lowry model, and only recently Cipriani et al. (2012) has used a parallel GA on a personal computer with dual-core processor for solving the transit network design problem.

Fig. 1 shows the procedures of DRGA. It can be seen that in each iteration new chromosomes are yielded by three processes: crossover, mutation and toll adjustment. Then, all the newly generated chromosomes are taken for evaluation, which possesses more than 90% of the total CPU time. As mentioned earlier, the evaluation process for each chromosome is conducted by different processors in the distributed computing system synchronously. Suppose that the total number of processors equals m, and all the newly generated chromosomes are evenly assigned to these m processors. Then, all the processors would work in parallel, which largely reduces the total execution time. After the evaluation, computational results for each newly generated chromosome are sent to the main processor for selection and stop test.

5. Numerical example

To numerically validate the proposed model and algorithm for the speed-based toll design problem, a network example is built based on the cordon-based ERP system in downtown Orchard Road in Singapore.

Fig. 2, downloaded from the website of the Singapore Land Transport Authority (2012), shows the charging locations on Orchard Road, and it is clear that at each entry link to the cordon a charge is made. Based on the map shown in Fig. 2, a grid network example with 33 nodes and 104 links is built as shown in Fig. 3. The pricing cordon is highlighted by a dotted ellipse, and all 12 entries are indicated by thick blue lines. The entry points are grouped into a set $\overline{A}$, as follows:

$$\overline{A} = \{24, 25, 27, 29, 34, 47, 79, 82, 84, 86, 88, 90\} \quad (30)$$

These entry links are used sequentially to build the chromosomes in the DRGA. The target range for the average speed of vehicles in the Orchard Road cordon has been chosen by the Singapore Land Transport Authority to be [20, 30] km/h. According to the speed-based toll design problem, the toll charges at each entry point must be adjusted to keep the average travel

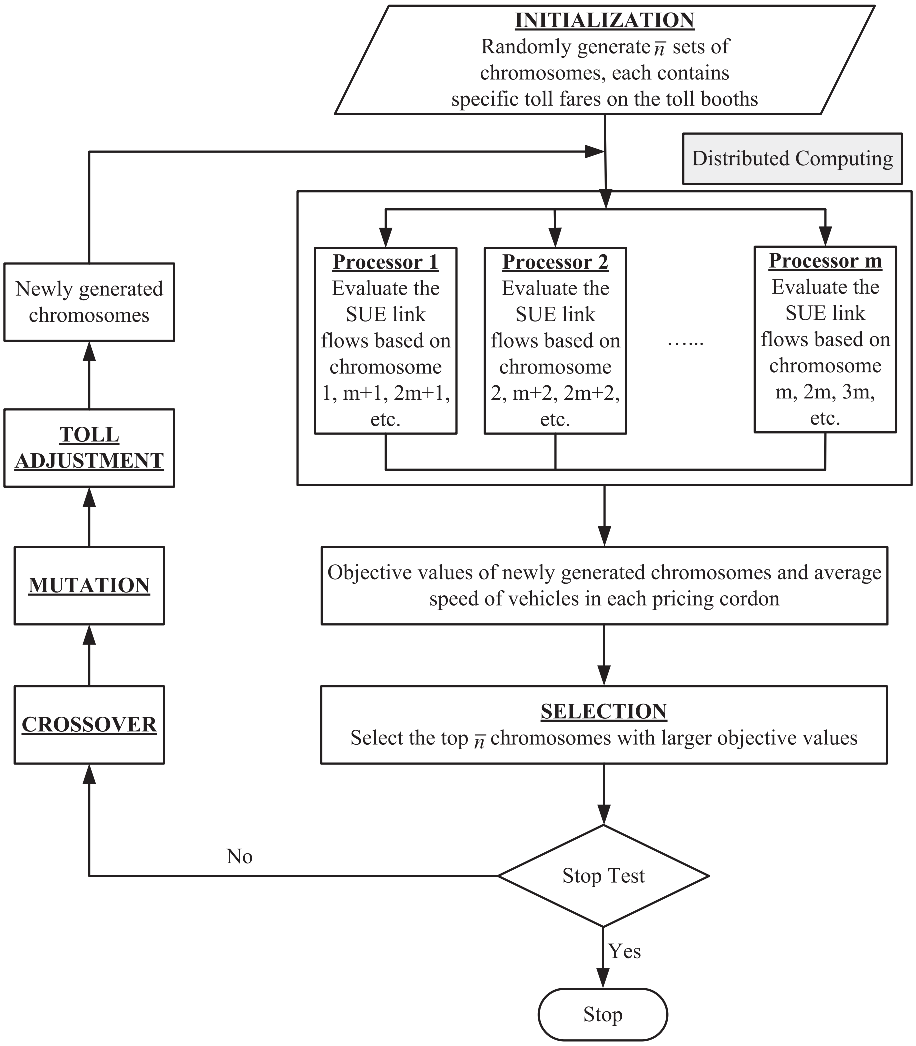

Fig. 1. Flowchart of distributed revised genetic algorithm.

speed within this range and, also, achieve the largest TSB. The increment π in the toll adjustment procedure (see Step 5 of the revised genetic algorithm in Section 4.1) is currently taken to be 1.0 Singapore Dollar (S$).

It is assumed that 12 OD pairs exist in this network. Table 1 shows the origin and destination nodes of each OD pair as well as the upper bound of its travel demand. The actual travel demand between OD pair $w \in W$ is assumed to be determined by the following function:

$$q_w = \bar{q}_w \times \exp(-0.001 \times S_w(\mathbf{v}, \boldsymbol{\tau})), \quad w \in W \tag{31}$$

The asymmetric link travel time function on link $a \in A$ is defined as follows (Bar-Gera, 2010):

$$t_a(\mathbf{v}) = t_a^0\left(1 + 0.15 \times \left(\frac{v_a + 0.5 v_{\hat{a}}}{1.5 h_a}\right)^4\right), \quad a \in A \tag{32}$$

where h_a is the capacity of the link flow and t_a^0 is the free-flow travel time. The values of h_a and t_a^0 on each link are shown in Table 2. We can see from Fig. 3 that most of the links in this network are accompanied by another link that goes in the opposite direction. Let $\hat{a}$ denote the opposite link to link a; then $v_{\hat{a}}$ in Eq. (32) denotes the flow on link $\hat{a}$. Accordingly, Eq. (32) implies that the travel time on each link is influenced by the flow on its opposite link as well as its own link flow, which makes the link travel time functions asymmetric.

In the context of the probit-based SUE principle, commuters' VOTs are assumed to have a normally distributed perception error on the link travel time, determined by Eq. (19). In this example, the value of the variance parameter β in Eq. (19) is

Fig. 2. Locations of ERP system on the orchard road of Singapore.

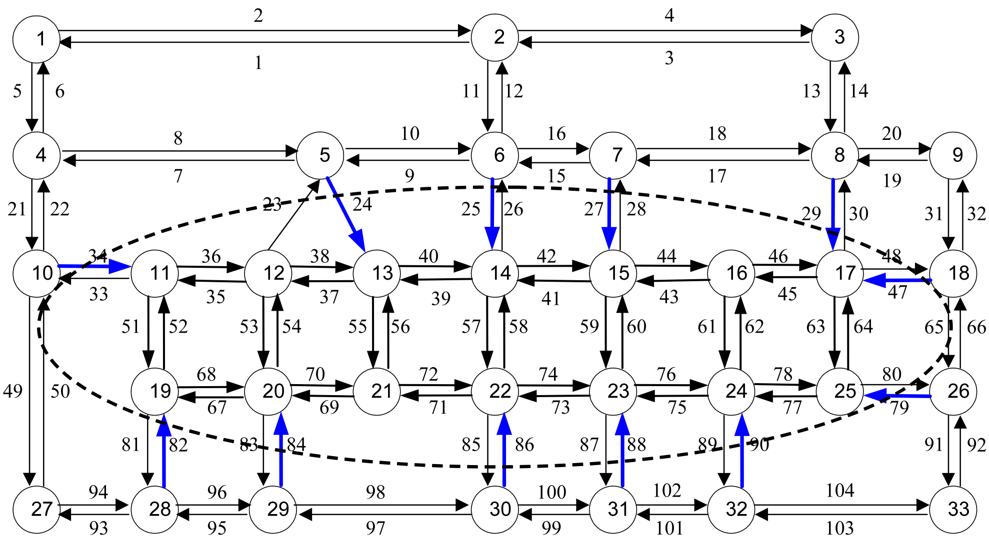

Fig. 3. Topological structure of the orchard road network.

taken to be 0.1. As mentioned earlier, commuters' VOT is assumed to be continuously distributed; we assume that the VOT is uniformly distributed, ranging from 18.0 to 72.0 S$ per h.

5.1. Simulation method for the average travel speed in each cordon

As per the speed-based toll design method, the toll charges at each entry point to the pricing cordon *i* should be adjusted based on the average journey speed of all the vehicles in that cordon during the decision period. Herein, the decision period is defined as 1 h during the morning peak time. In reality, after the implementation of any new toll charge pattern, the average journey speed of vehicles in the cordon can be obtained by a survey using probe vehicles. In this numerical example, the

Table 1
Parameters involved in the travel demand function for each OD pair.

OD pair w	Upper bound of travel demand $\bar{q}_w$ (vehicles/h)
$1 \rightarrow 33$	5000
$9 \rightarrow 1$	4000
$3 \rightarrow 27$	5000
$27 \rightarrow 9$	5000
$2 \rightarrow 29$	6000
$18 \rightarrow 28$	6000
$4 \rightarrow 24$	3000
$32 \rightarrow 14$	5000
$33 \rightarrow 3$	5000
$25 \rightarrow 4$	5000
$28 \rightarrow 6$	8000
$7 \rightarrow 23$	8000

corresponding average speed γ_i in cordon i is approximated by an area-wide speed-flow model proposed by Olszewski et al. (1995) for the downtown area of Singapore:

$$Q_i = 80.645\gamma_i(44.9 - 12.0\ln\gamma_i)^{1.563} - 2121.8, \quad i = 1, 2, \dots I \tag{33}$$

where Q_i denotes the total in-bound plus out-bound volume to cordon i, which equals the summation of the flows on all entry and exit points. With any given toll charge pattern $\boldsymbol{\tau} = (\tau_a, a \in \overline{A})^{\mathrm{T}}$, the equilibrium link flows on all the entry and exit points for cordon i can be obtained by solving a generalized probit-based SUE problem, as discussed in Section 3, and consequently the cordon's total inbound and outbound volume $Q_i(\tau)$, $i = 1, 2, \dots, I$ equals the flow on all the entry and exit points of the pricing cordon. Taking $Q_i(\tau)$, $i = 1, 2, \dots, I$ into Eq. (33) gives an estimate of the average travel speed $\gamma_i(\tau)$, and this is then used to adjust the toll charges at each entry point to cordon i.

5.2. Computing platform and performance measures

Before we talk about the computational results of using the DRGA to solve the speed-based toll design problem, the computing platform used and performance measures of the distributed computing method will be described briefly.

The computing platform used in this study is a high performance computing (HPC) system in the Civil and Environmental Engineering department at the National University of Singapore. This system has 60 computer nodes with distributed memory, and each node uses an Intel® Core i7 940 (Quad Core) processor, with a clock speed of 2.93 GHz, 256kB L2 cache per core and 8 MB L3 cache and 12 GB of 1333 MHz DDR3 RAM. These nodes are connected to an Ethernet Local Area Network via the 10G Myrinet technology and corresponding products, which allows a 10-Gigabit/s data delivery velocity. Regarding the software, an x64-based HPC Cluster Manager is installed on every node for configuring, deploying and managing the cluster, and data communication as well as job assignment is conducted by means of the Message Passing Interface (MPI) (Gropp et al., 1999). The MPI protocol can support both point-to-point and collective communication. All programs used for this paper are coded in FORTRAN 90, for which MPI acts as a function library.

The DRGA is tested under different scenarios, using different numbers of processors in the distributed computing system. The execution time as well as a well-known performance measure, known as Speed-Up (Nagel and Rickert, 2001; Wong et al., 2001; Liu and Meng, 2011), are used to evaluate its performance under each scenario. The value of Speed-Up can be calculated as follows:

$$S(m) = \frac{T_1}{T_m} \tag{34}$$

where T_1 denotes the execution time when using only one processor, and T_m the execution time when m processors are used.

5.3. Computational results of the distributed revised genetic algorithm

For the DRGA, the values for both the population and generation are chosen to be 50. The computation is terminated after 50 generations, which is taken as a stop criterion. The mutation and crossover rates are set to be 0.01 and 0.25, respectively. The synchronized evaluation procedure solves a generalized probit-based SUE problem for each newly generated chromosome using the CA method. The sample size at each stage of the Monte Carlo simulation, as discussed in Section 3.2, is chosen to be 100 for stage 1 and 1000 for stage 2, based on some empirical tests. In addition, the penalty parameter c in Eq. (28) is set to be 1.0×10^6.

The upper bound $\bar{\tau}$ and the lower bound $\underline{\tau}$ for the positive toll charges at each entry point are taken to be 10.0 S$ and 0.0 S$, respectively. An initial generation of the DRGA is then produced by independently selecting a random number in the

Table 2
Data for the link attributes.

Link no.	Start node	End node	Free-flow travel time t_a^0 (s)	Capacity h_a (vehicles/h)
1	2	1	60	5400
2	1	2	60	5400
3	3	2	100	5400
4	2	3	100	5400
5	1	4	40	5400
6	4	1	40	5400
7	5	4	90	3600
8	4	5	90	3600
9	6	5	42	3600
10	5	6	42	3600
11	2	6	80	7200
12	6	2	80	7200
13	3	8	72	3600
14	8	3	72	3600
15	7	6	160	1800
16	6	7	160	1800
17	8	7	120	1800
18	7	8	120	1800
19	9	8	60	3600
20	8	9	60	3600
21	4	10	80	5400
22	10	4	80	5400
23	12	5	55	3600
24	5	13	55	3600
25	6	14	80	7200
26	14	6	80	7200
27	7	15	120	1800
28	15	7	120	1800
29	8	17	60	3600
30	17	8	60	3600
31	9	18	90	3600
32	18	9	90	3600
33	11	10	16	5400
34	10	11	16	5400
35	12	11	40	5400
36	11	12	40	5400
37	13	12	24	5400
38	12	13	24	5400
39	14	13	48	5400
40	13	14	48	5400
41	14	15	40	5400
42	15	14	40	5400
43	16	15	12	5400
44	15	16	12	5400
45	17	16	60	5400
46	16	17	60	5400
47	18	17	60	5400
48	17	18	60	5400
49	10	27	20	5400
50	27	10	20	5400
51	11	19	24	1800
52	19	11	24	1800
53	12	20	20	5400
54	20	12	20	5400
55	13	21	30	3600
56	21	13	30	3600
57	14	22	12	7200
58	22	14	12	7200
59	23	15	12	3600
60	15	23	12	3600
61	16	24	12	3600
62	24	16	12	3600
63	17	25	14	3600
64	25	17	14	3600
65	18	26	20	5400
66	26	18	20	5400
67	20	19	80	1800
68	19	20	80	1800

Table 2 (*continued*)

Link no.	Start node	End node	Free-flow travel time t_a^0 (s)	Capacity h_a (vehicles/h)
69	20	21	20	5400
70	21	20	20	5400
71	22	21	48	5400
72	21	22	48	5400
73	23	22	20	5400
74	22	23	20	5400
75	24	23	28	5400
76	23	24	28	5400
77	25	24	60	5400
78	24	25	60	5400
79	26	25	60	5400
80	25	26	60	5400
81	19	28	24	1800
82	28	19	24	1800
83	20	29	12	5400
84	29	20	12	5400
85	22	30	18	7200
86	30	22	18	7200
87	23	31	16	3600
88	31	23	16	3600
89	24	32	18	5400
90	32	24	18	5400
91	26	33	60	5400
92	33	26	60	5400
93	28	27	20	5400
94	27	28	20	5400
95	29	28	40	5400
96	28	29	40	5400
97	30	29	90	3600
98	29	30	90	3600
99	31	30	30	3600
100	30	31	30	3600
101	32	31	60	3600
102	31	32	60	3600
103	33	32	96	5400
104	32	33	96	5400

range [0.0, 10.0] to each gene of the chromosomes. It should be pointed out that, due to the toll adjustment process (see Step 5 in Section 4.1), the toll charges may fall outside the range of [0.0, 10.0] S$.

Fig. 4 shows the convergence trend of the DRGA, and provides the maximal value of the objective functions among all the chromosomes in each generation. Table 3 indicates the resultant optimal toll charges at each entry point to the Orchard Road cordon, denoted by $\boldsymbol{\tau}^*$. The corresponding TSB (in terms of Eq. (28)) and the average travel speed $\gamma_i(\boldsymbol{\tau}^*)$ in the Orchard Road cordon are 4.81×10^7 and 23.3 km/h, respectively. We can see that $\gamma_i(\boldsymbol{\tau}^*)$ is located within the target range [20, 30] km/h chosen by the Singapore Land Transport Authority.

To see the full impact of the toll charges on the network conditions, two additional tests are conducted for the cases with null toll (toll charges all equal to zero) and maximum tolls (the upper bound 10.0 S$ is levied at each entry point). The results

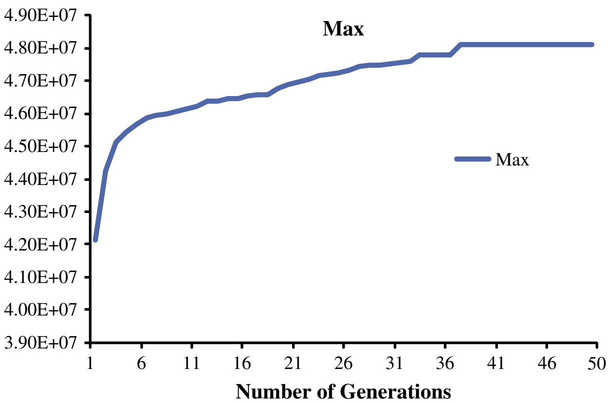

Fig. 4. Convergence trend of DRGA.

Table 3
Resultant toll charges on each entry.

Entry no.	24	25	27	29	34	47
Optimal toll charge (S$)	1.6	0.9	2.4	3.9	4.0	1.1
Entry no.	79	82	84	86	88	90
Optimal toll charge (S$)	5.3	3.9	1.3	4.0	5.6	2.1

Table 4
Execution time and speed-up with different number of processors.

No. of processors	Execution time (s)	Speed-up
1	107580	1
2	56,736	1.896151
3	41,330	2.602952
4	37,116	2.89848
5	28,836	3.730753
6	27,427	3.922412
7	26,458	4.066067
8	22,950	4.687582
9	20,399	5.273788
10	19,106	5.630692
15	14,501	7.418799
20	11,156	9.643241
25	10,080	10.67262
30	9826	10.9485

show that, for the untolled case, the TSB value is -5.17×10^7 and the average speed in the cordon is 10.1 km/h; for the case with the maximum toll, the TSB value is 2.56×10^7 and the average speed is 34.2 km/h. The computational results for these two extreme cases verify that cordon-based toll charges inherently influence the network users' route choices and thus can mitigate traffic congestion within the cordon area. For the untolled case, an average speed of 10.1 km/h implies a congested road condition, which is much worse than the expectation of the network authorities. In the case of maximum toll charges, a fast average speed of 34.2 km/h implies that quite a small number of vehicles are traveling in the area, which is a waste of the road resources. The TSB value for the untolled case is negative due to the high penalty cost of an unacceptable average travel speed.

The performance of the DRGA will be affected by the number of processors used. Hence, a sensitivity test is conducted to determine the impact on the total execution time as well as the value of Speed-Up. The results are shown in Table 4. It can be seen that, when only one processor is used for the calculation T_1, the execution time is as large as 107,580 s, that is approximately 30 h, which is beyond an acceptable level. The execution time, however, is sharply reduced when more processors are used. When 30 processors are used, the computation is accelerated by nearly 11 times, with an execution time of around 2.7 h.

To get a better view of the trend in this sensitivity test, the values of Speed-Up for different numbers of processors are indicated in Fig. 5. An interesting phenomenon shown by Fig. 5 is that, when the number of processors used is less than 10, the value of Speed-Up increases linearly, but the increase from each additional processor reduces when more than 10

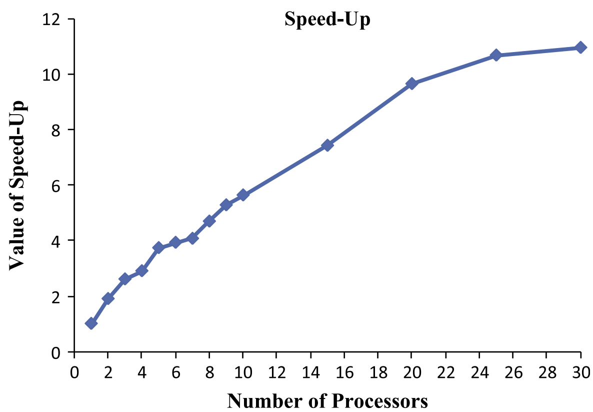

Fig. 5. Value of speed-up in terms of different number of processors.

processors are used. This phenomenon can be ascribed to two reasons: (a) When a new processor is added, there should be a trade-off between its marginal computation effort and the marginal cost resulting from the additional data communication load. Yet, thanks to the advanced 10G Myrinet technology adopted in the distributed computing system, the total data communication time is quite trivial, thus each additional processor can fully contribute to speeding up the computation, and this results in an approximately linear increase in Speed-Up. (b) In each generation of the DRGA, the number of newly generated chromosomes varies from 10 to 40. If this number is less than the number of processors used, the redundant processors will be idle, so they cannot contribute to speeding up the calculation. Thus, when the number of processors is larger than 10, the total idle time will increase dramatically, and the performance of the distributed computation will deteriorate.

It should be pointed out that, apart from cordon-based pricing, the ERP system in Singapore also features link-based tolls on some arterial roads and expressways, where the same rule is adopted for the toll setting. The proposed methodology in this paper is also applicable to this type of link-based toll system, if we take the arterial roads (or expressways) as a special case of a pricing cordon: the cordon area only contains the charging portion of the arterial road, maybe only one link, and the charging location is not the entry link but one link of this arterial road.

6. Conclusions

This paper deals with the speed-based toll design problem for a cordon-based congestion pricing scheme, taking an improvement of the traffic conditions in the cordon area as an objective. Average travel speed is taken as an indicator of the traffic conditions, and a target range for the average travel speed is predefined. Any toll charge pattern that keeps the average travel speed within this target range is regarded as acceptable. A MPEC model is proposed to determine the optimal toll charge pattern, among the acceptable ones, that produces the maximal TSB. The MPEC model takes a fixed-point model, formulated for the commuters' route choice problem, as a constraint. This route choice problem is, by nature, a probit-based SUE problem with continuously distributed VOT, elastic demand and asymmetric link travel time functions.

A DRGA is then proposed for solving this speed-based toll design problem, based on a distributed computing system. The results show that the DRGA can successfully find a toll charge pattern that keeps the average travel speed within the target range while maximizing TSB. The numerical example further indicates that the computation can be speeded up by more than ten times through the use of multiple processors.

This study represents an initial step towards including the traffic conditions in the CBD area as well as the issue of average travel speed into the congestion pricing toll design problem. A promising research topic would be an in-depth and more practical investigation into the effect of a given toll charge pattern on the average travel speed in the cordon area, to provide a more proper flow average-speed relationship function. Furthermore, it is also necessary to extend the current work to the cases of dynamic traffic assignment, multiple vehicle types and Pareto-improving the toll design, among other things.

Further efforts are also required to investigate the impacts of different distributions of the VOT on the optimal toll charge pattern. A calibration of the VOT distribution based on practical survey data would be of considerable significance to this research topic.

References

Allsop, R.E., 1974. Some possibilities of using traffic control to influence trip distribution and route choice. In: Buckley, D.J. (Ed.), Proceedings of the 6th International Symposium on Transportation and Traffic Theory, pp. 345–375.

Bar-Gera, H., 2010. Transportation Network Test Problems. <http://www.bgu.ac.il/~bargera/tntp/> (accessed 08.02.12).

Bellei, G., Gentile, G., Papola, N., 2002. Network pricing optimization in multi-user and multimodal context with elastic demand. Transportation Research Part B 36, 779–798.

Cantarella, G.E., 1997. A general fixed-point approach to multimode multi-user equilibrium assignment with elastic demand. Transportation Science 31 (2), 107–128.

Cantarella, G.E., Binetti, M.G., 1998. Stochastic equilibrium traffic assignment with value-of-time distributed among user. International Transactions of Operational Research 5 (6), 541–553.

Chen, M., Bernstein, D.H., 2004. Solving the toll design problem with multiple user groups. Transportation Research Part B 38, 61–79.

Chiou, S.W., 2005. Bilevel programming for the continuous transport network design problem. Transportation Research Part B 39 (4), 361–383.

Cipriani, E., Gori, S., Petrelli, 2012. Transit network design: a procedure and an application to a large urban area. Transportation Research Part C 20 (1), 3–14.

Clark, S.D., Watling, D.P., 2002. Sensitivity analysis of the probit-based stochastic user equilibrium assignment model. Transportation Research Part B 36, 617–635.

Connors, R.D., Sumalee, A., Watling, D.P., 2007. Sensitivity analysis of the variable demand probit stochastic user equilibrium with multiple user-classes. Transportation Research Part B 41, 593–615.

Daganzo, C.F., Sheffi, Y., 1977. On stochastic models of traffic assignment. Transportation Science 11 (3), 253–274.

de Palma, A., Lindsey, R., 2011. Traffic congestion pricing methodologies and technologies. Transportation Research Part C 19 (6), 1377–1399.

Dial, R.B., 1996. Bicriteria traffic assignment: basic theory and elementary algorithm. Transportation Science 30, 93–111.

Dial, R.B., 1997. Bicriterion traffic assignment: efficient algorithm plus examples. Transportation Research Part B 31, 357–379.

Eliasson, J., 2009. A cost-benefit analysis of the Stockholm congestion charging system. Transportation Research Part A 43 (4), 468–480.

Gen, T., Cheng, R., 1997. Genetic Algorithms and Engineering Design. John Wiley & Sons, Inc.

Goldberg, D., 1989. Genetic Algorithms in Search, Optimization and Machine Learning. Addison-Wesley, Reading, MA.

Gropp, W., Lusk, E., Skjellum, A., 1999. Using MPI: Portable Parallel Programming with the Message-Passing Interface. The MIT Press.

Han, D., Yang, H., 2008. The multi-class, multi-criterion traffic equilibrium and the efficiency of congestion pricing. Transportation Research Part E 44 (5), 753–773.

Han, D., Yang, H., 2009. Congestion pricing in the absence of demand functions. Transportation Research Part E 45 (1), 159–171.

Huang, H.J., Yang, H., 1996. Optimal variable road-use pricing on a congested network of parallel routes with elastic demand. In: Lessort J.B. (Ed.), Transportation and Traffic Theory. Elsevier Science, pp. 479–500.

Lam, T.C., Small, K.A., 2001. The value of time and reliability: measurement from a value pricing experiment. Transportation Research Part E 37, 231–251.

Lawphongpanich, S., Yin, Y., 2012. Nonlinear pricing on transportation networks. Transportation Research Part C 20 (1), 218–235.

Lawphongpanich, S., Hearn, D.S., Smith, M.J., 2006. Mathematical and Computational Model for Congestion Charging. Springer.

Leurent, F., 1993. Cost versus time equilibrium over a network. European Journal of Operational Research 71, 205–221.

Lewis, N.C., 1993. Road Pricing: Theory and Practice. Thomas Telford, London.

Li, M.Z.F., 1999. Estimating congestion toll by using traffic count data – Singapore's Area Licensing Scheme. Transportation Research Part E 35, 1–10.

Li, M.Z.F., 2002. The role of speed-flow relationship in congestion pricing implementation with an application to Singapore. Transportation Research Part B 36, 731–754.

Liu, Z., Meng, Q., 2011. Distributed computing approaches for large-scale probit-based Stochastic User Equilibrium problems. Journal of Advanced Transportation. http://dx.doi.org/10.1002/atr.177.

Liu, Z., Meng, Q., 2012. Modelling transit-based park-and-ride services on a multimodal network with congestion pricing schemes. International Journal of Systems Science. http://dx.doi.org/10.1080/00207721.2012.743617.

Maher, M.J., Hughes, P.C., 1997. A probit-based stochastic user equilibrium assignment model. Transportation Research Part B 31 (4), 341–355.

Maher M.J., Zhang X., 2000. Formulation and algorithms for the problem of stochastic user equilibrium assignment with elastic demand. In: 8th EURO Working Group Meeting on Transportation, Rome, September 2000.

Maher, M., Stewart, K., Rosa, A., 2005. Stochastic social optimum traffic assignment. Transportation Research Part B 39, 753–767.

Mayet, J., Hansen, M., 2000. Congestion pricing with continuously distributed values of time. Journal of Transport Economics and Policy 34, 359–370.

McDonald, J.F., 1995. Urban highway congestion. An analysis of second-best tolls. Transportation 22 (4), 353–369.

Meng, Q., Liu, Z., 2011. Mathematical models and computational algorithms for probit-based asymmetric stochastic user equilibrium problem with elastic demand. Transportmetrica. http://dx.doi.org/10.1080/18128601003736026.

Meng, Q., Liu, Z., Wang, S., 2012. Optimal distance tolls under congestion pricing and continuously distributed value of time. Transportation Research Part E 48, 937–957.

Meng, Q., Yang, H., Bell, M.G.H., 2001. An equivalent continuously differentiable model and a locally convergent algorithm for the continuous network design problem. Transportation Research Part B 35 (1), 83–105.

Nagel, K., Rickert, M., 2001. Parallel implementation of the TRANSIMS micro-simulation. Parallel Computing 27, 1161–1639.

Nie, Y., Liu, Y., 2010. Existence of self-financing and Pareto-improving congestion pricing: impact of value of time distribution. Transportation Research Part B 44, 39–51.

Olszewski, P., Xie, L., 2005. Modelling the effects of road pricing on traffic in Singapore. Transportation Research Part A 39, 755–772.

Olszewski, P., Fan, H.S.L., Tan, Y.-W., 1995. Area-wide traffic speed-flow model for the Singapore CBD. Transportation Research Part A 29 (4), 273–281.

Phang, S.-Y., Toh, R.S., 1997. From manual to electronic road congesting pricing: the Singapore experience and experiment. Transportation Research Part E 33 (2), 97–106.

Rosa, A., Maher, M.J., 2002. Algorithms for solving the probit path-based SUE traffic assignment problem with one or more user classes. In: Transportation and Traffic Theory in the 21st Century. Proceedings of the 15th ISTTT, pp. 371–392.

Santos, G., 2008. The London congestion charging scheme, 2003–2006. In: Richardson, H.W., Bae, C-H.C., (Eds.), Road congestion pricing in Europe – Implications for the United States. Edward Elgar Publishing, Inc.

Sheffi, Y., 1985. Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Models. Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Sheffi, Y., Powell, W.B., 1981. A comparison of stochastic and deterministic traffic assignment over congested networks. Transportation Research Part B 15 (1), 53–64.

Singapore Land Transport Authority Website, 2012. <http://www.lta.gov.sg/content/lta/en.html> (accessed 04.08.12).

Small, K.A., Verhoef, E.T., 2007. The Economics of Urban Transportation. Routledge, London.

Small, K.A., Winston, C., Yan, J., 2005. Uncovering the distribution of motorists' preferences for travel time and reliability. Econometrica 73 (4), 1367–1382.

Sumalee, A., Xu, W., 2011. First-best marginal cost toll for a traffic network with stochastic demand. Transportation Research Part B 45, 41–59.

Suwansirikul, C., Friesz, T.L., Tobin, R.L., 1987. Equilibrium decomposed optimization: a heuristic for the continuous equilibrium network design problems. Transportation Science 21 (4), 254–263.

Verhoef, E.T., Small, K.A., 2004. Product differentiation on roads: constrained congestion pricing with heterogeneous users. Journal of Transport Economics Policy 38 (1), 127–156.

Wong, S.C., Wong, C.K., Tong, C.O., 2001. A parallelized genetic algorithm for the calibration of Lowry model. Parallel Computing 27, 1523–1536.

Xiao, F., Yang, H., 2008. Efficiency loss of private road with continuously distributed value-of-time. Transportmetrica 4, 19–32.

Yang, H., 1997. Sensitivity analysis for the elastic-demand network equilibrium problem with applications. Transportation Research Part B 31 (1), 55–70.

Yang, H., Huang, H.-J., 1998. Principle of marginal-cost pricing: how does it work in a general network? Transportation Research Part A 32, 45–54.

Yang, H., Huang, H.-J., 2005. Mathematical and Economic Theory of Road Pricing. Elsevier Ltd..

Yang, H., Kong, H.Y., Meng, Q., 2001. Value-of-time distributions and competitive bus services. Transportation Research Part E 37 (6), 411–424.

Yang, H., Meng, Q., Lee, D.-H., 2004. Trial-and-error implementation of marginal-cost pricing on networks in the absence of demand functions. Transportation Research Part B 38, 477–493.

Yang, H., Xu, W., He, B.S., Meng, Q., 2009. Road pricing for congestion control with unknown demand and cost functions. Transportation Research Part C 18 (2), 157–175.

Yang, H., Xu, W., Heydecker, B., 2010. Bounding the efficiency of road pricing. Transportation Research Part E 46 (1), 90–108.

Zhao, Y., Kockelman, K.M., 2006. On-line marginal-cost pricing across networks: incorporating heterogeneous users and stochastic equilibria. Transportation Research Part B 40, 424–435.