Modeling no-notice mass evacuation using a dynamic traffic flow optimization model

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This paper presents a network transformation and demand specification approach for no-notice evacuation modeling. The research is aimed at formulating the Joint Evacuation Destination–Route-Flow-Departure (JEDRFD) problem of a no-notice mass evacuation into a system optimal dynamic traffic assignment model. The proposed network transformation technique permits the conversion of a typical transportation planning network to an evacuation network configuration in which a hot zone, evacuation destinations, virtual super-safe node and connectors are established. Combined with a demand specification method, the JEDRFD problem is formulated as a single-destination cell-transmission-model-based linear programming model. The advantage of the proposed model compared with prior studies in the literature is that the multi-dimensional evacuation operation decisions are jointly obtained at the optimum of the JEDRFD model. The linear single-destination structure of the proposed model implies another advantage in computational efficiency. A numerical example is given to illustrate the modeling procedure and solution properties. Real-time operational issues and data requirements are also discussed.

Keywords: No-notice, mass evacuation, emergency management, dynamic traffic assignment, linear programming, cell transmission model

1. Introduction

Mass evacuation is required when a natural or man-made extreme event (e.g., hurricane, flooding, hazmat release, or terrorist attack) strikes or threatens a populated area exposing it to an immediate or imminent life-threatening condition. Undertaking this difficult task primarily relies on the efficient coordination and utilization of roadway capacity, traffic management equipment and available emergency response resources. The evacuation strategy may vary based on two types of disaster-induced evacuation characterizations: short-notice or no-notice disasters. Short-notice disasters are those that have a desirable lead time of between 24–72 hours (Wolshon et al., 2002) allowing emergency management agencies (EMAs) to determine alternate evacuation strategies a priori based upon the expected spatial-temporal impacts of the disaster. Examples of short-notice disasters are events such as hurricanes, flooding and wild fires.

Conversely, a no-notice evacuation takes place when any large and unexpected incident occurs. The evacuation that takes place immediately after the occurrence of a disaster event is defined as a “no-notice evacuation” (Anon, 2005). When a no-notice disaster occurs requiring a mass evacuation, a preconceived evacuation plan can be immediately put in action. Traffic control and routing strategies need to be rapidly and frequently updated according to unfolding traffic conditions. An EMA is often faced with control and routing strategies that typically involve four critical operational decisions, including: (i) decide where to evacuate people (destinations); (ii) decide on the best routes to take (route); (iii) determine how to regulate flow rates on these routes (traffic assignment); and (iv) determine the rate at which evacuees need to be permitted to enter the network from different areas of the region (phased departure schedule). Obviously, these decisions are interdependent and making such decisions simultaneously and in a coherent manner is methodologically and computationally challenging.

Methodological challenges arise since typical metropolitan surface transportation systems consist of various highways of different functional classes that are interconnected with varying topology and connectivity. Deciding on an optimal evacuation destination-route-flow schedule and departure time requires a systematic approach that fully

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utilizes the advantage of optimization techniques. The complexity of this problem can be illustrated with the layout of the evacuation routes for New Orleans, LA as shown in Fig. 1. All the quadrants (except the south) surrounding the city are designated as possible evacuation directions. For each evacuation quadrant, multiple evacuation destinations may exist. Evacuation destinations are defined as the final locations at which evacuees are considered to be safe and hence they define the perimeter of the evacuation network of interest. For each evacuation destination, multiple evacuation routes may exist. Each route may exhibit an individual geometric configuration and traffic flow capacity. Determining how to evacuate evacuees through the optimal utilization of all the routes to all possible destinations requires a systematic consideration of the flows and capacity of each route and its constituent links.

Dynamic network flow models can be used to address this problem. For this evacuation application, it is particularly important to incorporate the explicit modeling of traffic flow dynamics with the optimal multidimensional destination-route-traffic assignment-departure schedule decisions into a unified model so that the optimal solutions are consistent with traffic flow dynamics. This model should also have a simple structure that can be solved efficiently so that the optimal solution can be obtained soon after the occurrence of the disaster. To the authors’ best knowledge, models meeting the above criteria are currently nonexistent in the literature or practice. Over the past decade, a number of models have been developed to address in emergency evacuation planning to mitigate disasters ranging from nuclear plant failures to hurricanes. These models, however, are more descriptive in nature (i.e., predict how evacuation traffic flow will evolve with minimal notion of control) than prescriptive (i.e., determine the optimal evacuation routing strategies to achieve the optimal evacuation goal). For those studies that focus on the optimal evacuation control, the evacuation origin-destination information is often assumed to be given. How to obtain the optimal evacuation demand has rarely been discussed.

Studies in the 1980s focused on nuclear-site-related evacuations (no-notice disaster) principally driven by the Three Mile Island incident which occurred in 1979 (Urbanik and Desrosiers, 1981; Sheffi et al., 1982; Anon, 1984; Han, 1990). These studies resulted in macroscopic simulation models such as DYNEV (Anon, 1984). (These are models that describe vehicular traffic as fluid and use macroscopic measures such as flow, density, average speeds, etc. as compared to the representation of individual driver behavior that is used in microscopic traffic simulation models.) In response to the severe hurricanes that hit the US in the 1990s, evacuation research has tended to emphasize hurricane evacuation (short-notice disaster) (Hobeika and Jamei, 1985; Witte, 1995; Anon, 1999, 2000; Wolshon, 2001). Examples of models developed to address hurricane evacuation include MASSVAC, OREMS and ETIS.

MASSVAC uses macroscopic traffic flow models to forecast hurricane evacuation performance (Hobeika and Jamei, 1985; Hobeika et al., 1985). The Oak Ridge Evacuation Modeling System (OREMS) is a software package that can be used to model evacuation operations and planning and management scenarios for a variety of disasters (Anon, 1999). The OREMS requires the input of time-dependent evacuation demand data with or without specification of the evacuation destinations. If the evacuation demand is
No-notice mass evacuation modeling

This paper is structured as follows. Section 2 discusses the major decisions associated with no-notice evacuation faced by an EMA. Section 3 presents the network transformation and evacuation demand specification techniques for modeling the no-notice mass evacuation problem. The formulation of the Joint Evacuation Destination-Route-Flow-Departure (JEDRFD) schedule problem in terms of related theories such as the CTM is also presented in Section 3. Section 4 discusses a numerical case study in which the proposed approach is applied to an example network. Section 5 provides concluding remarks on the potential of this proposed method and possible future research directions.

2. No-notice mass evacuation operation objectives and decisions

In short-notice and no-notice evacuations, the perimeter of the impacted area, otherwise termed the hot zone, could be time dependent or time invariant depending on the characteristics of the disaster. Some relatively common disasters such as hurricanes, flooding, or an airborne hazmat release have dynamic hot zone perimeters due to their time-varying spatial trajectories. This trajectory information can be estimated using meteorological approaches (hurricane),
hydraulic analysis (flooding) or plume dispersion modeling techniques (airborne hazmat release). When such capabilities are available, it is reasonable to assume that the hot zone perimeter information can be used as a model input.

However, this information may not always be available when a disaster strikes. In this case, estimating the hot zone perimeter utilizing expert opinion and/or predefined EMAs operational policies becomes necessary. Under such a circumstance, the definition of the hot zone is typically at the EMA’s discretion and will be based on their overall situation assessment. For example, during the 9/11 incident, the hot zone was defined as the entire Manhattan area and all residents were asked to leave the hot zone despite only a limited number of building having collapsed or been damaged. Thus, assuming that the hot zone perimeter can be used as a known model input is consistent with actual practice. Therefore, in the rest of this paper we assume that the hot zone information is available soon after an incident occurs; being obtained either through the output of other models or expert opinion or policies.

The designation of the target horizon over which the model is solved also depends on the nature of the disaster. In chemical spills or nuclear radiation leakage scenarios, there is an allowable human exposure time beyond which fatalities and/or permanent physical damage becomes inevitable. This can be estimated by the meteorological estimation of toxic substance dispersion. In the case of flooding, the operational horizon may be driven by the estimated time in which the hot zone becomes inundated. In either case, the operational horizon can be treated as a model input.

### 2.1. Evacuation objectives

The evacuation objectives and decisions faced by an EMA under no-notice and short-notice situations differ due to the nature of the disasters. Typically, short-notice events are relatively predictable; forecasting the occurrence of the event is feasible. In the case of a hurricane, weather services are able to track the storm system days prior to the landing. The objective of such an evacuation operation is to select a set of destinations, evacuation routes, and evacuation schedule to minimize the evacuation time or network clearance time, i.e., the time until the last vehicle leaves the affected area. In practice, the evacuation time defines the lead time required to evacuate the population and sets a target deadline for an evacuation order.

Evacuation under a no-notice situation aims at either maximizing the number of people exiting the hot zone or minimizing causalities or exposure within the hot zone given the target evacuation horizon. Unlike the short-notice event in which the disaster trajectory can be reasonably predicted, the location and time of a no-notice disaster occurrence is unpredictable. Thus, the amount of time allowed to develop an evacuation strategy is very limited. In this case, the notion of “network clearance time” becomes less relevant than maximizing the total number of safe evacuees by the target evacuation horizon. In other words, the primary concern of a no-notice evacuation may not necessarily be to reduce the network clearance time, but instead to maximize the safe evacuation numbers or minimize the total amount of casualties or exposure to the threat. From a modeling point of view, these two scenarios require distinct objectives for the optimization model to be built.

### 2.2. Evacuation decisions

There is a myriad of operational decisions that need to be made during an evacuation. These decisions range from coordination between different public and private agencies, logistics of critical personnel and equipment, disseminating information, and implementing traffic control strategies. The common critical evacuation decisions include: (i) the selection of available safe destinations, which may include areas outside the hot zone or specific shelters within the hot zone; (ii) the determination of evacuation routes and the optimal traffic volume that can be assigned to each route; and (iii) the determination of optimal departure schedules for evacuees dispersed through the transportation network in order to minimize the overall clearance time. These three dimensions of mutually coupling decisions will each affect the effectiveness of the overall evacuation effort and an ideal evacuation model should simultaneously obtain optimal solutions for all three decision dimensions. The no-notice evacuation model which will be described in the next section is aimed at addressing this challenge.

### 3. JEDRFD schedule optimization modeling approach

The discussions in this section focus on techniques to model the aforementioned operational decisions into a SO DTA model. The modeling approach consists in defining both the network transformation and evacuation demand specification. The network transformation method is first discussed followed by the demand specification.

The network transformation method first requires that the network be defined into zones. Typical zoning schemes are the traffic analysis zones (TAZs) used for transportation planning purposes, or zip codes. If a TAZ approach is used, then each TAZ needs to be identified (see Fig. 2(a)) as an evacuation zone (hot zone), intermediate zone (warm zone), or safe zone for the evacuation operation purpose. Defining the perimeter of a hot zone requires knowledge of the spatial-temporal progression of the disaster, target evacuation horizon and possibly the terrain barrier characteristics. As previously discussed, several ways exist to define the hot zone. In this paper, the perimeter of the hot zone is assumed given from an external mechanism.

The next step is to keep the topology of the hot and warm zones unchanged and designate all boundary nodes in the safe zone as possible evacuation destinations. Evacuees are considered to be safe on their arrival at these destinations.
It should be noted that by utilizing this hot/safe zone definition, the evacuation destinations become enumerable because only a finite number of nodes will be located at the boundary of the safe zone. Furthermore, a super-safe (sink) node with an infinite capacity is introduced. All the evacuation destinations are connected to this super-safe node through virtual connectors with an infinite capacity. One additional TAZ containing only the super-safe node also needs to be created. All evacuation flows will be sent to this super-safe node by way of the evacuation destinations. In other words, all the hot zone boundary nodes act as possible gateways for evacuees to reach the super-safe node. However, only those destination nodes located on routes assigned with flows by the DTA model are considered to be the optimal destinations. Furthermore, a virtual source node is created for each evacuation origin node. An origin node can be any physical node in the hot zone. Each source node needs to connect to its corresponding evacuation origin node through a virtual connector (see Fig. 2(b)). A source node is used to store the total evacuation flow for the source node’s corresponding origin node at the beginning of the evacuation. The discharge of the evacuation flow into the origin node throughout the evacuation time period will be solved by the JEDRFD model presented in Section 3.2. Loading all evacuation flows at time 0 has a physical meaning. In a no-notice evacuation all evacuees would like to leave immediately; loading them into the source node at the beginning of the evacuation reflects such a reality. The optimal departure of evacuation flows, however, needs to be regulated so that evacuees do not overload the transportation network and cause further problems.

The final transformed single-destination network of interest consists of only the hot zone, evacuation destinations, super-safe node and the virtual connectors between the evacuation destinations and the super-safe node. The rest of the actual network can be removed as far as modeling is concerned. At this point, the network transformation is complete. The transformed network is effectively a multiple-source single-sink network.

This network transformation technique is general and flexible enough to cope with most evacuation scenarios with various disaster trajectory and hot zone definitions. For instance, Fig. 2(b) illustrates the scenario in which directional evacuation is ordered from the left to the right side of the network in response to a hurricane or airborne chemical release emergency. On the other hand, Fig. 2(c) illustrates the scenario in which the evacuation follows a radial pattern which may be encountered in a building collapse emergency.

The time-dependent demand is defined as the number of vehicular trips from an origin zone to a particular destination zone for all Origin-Destination (OD) zone pairs in the network. Time-dependent demand is usually represented by multiple zonal trip matrices, with each matrix representing the zonal OD trips for each time interval of interest. The proposed modeling approach does not require this type of demand matrix. The originating demand, determined through zonal population estimates, is loaded on each corresponding source node at the beginning of the
evacuation, as later shown in Equation (10) and Tables 3 and 4. The time-dependent optimal outgoing evacuation flow rate at the origin nodes and the time-dependent incoming flow rate at the super-safe node are then solved using the JEDRFD model. In other words, our proposed approach solves for the optimal time-dependent evacuation OD demand instead of requiring the OD demand to be a model input. This demand specification treatment is the major difference between our approach and prior studies that use DTA for ordinary transportation planning and operation applications.

At this point, the network transformation and demand specification is complete, but further integration with a SO DTA model is needed to formulate the JEDFRF problem. In this research, without loss of generality, the proposed modeling approach is integrated with a CTM-based Linear Programming (LP) formulation although other SO DTA models can also be used. The following sections first give a brief overview of the CTM model followed by the JEDRFD formulation.

3.1. The CTM

The CTM (Daganzo, 1994, 1995) is an innovative transformation of the Lighthill-Whitham-Richards (LWR) (Lighthill and Whitham, 1955; Richards 1956) hydrodynamic traffic flow model that simplifies the difference equations by assuming a piecewise linear relationship between the flow and density at the cell level. One can view the CTM as a macroscopic traffic simulation model. The model accurately describes traffic propagation on street networks and captures traffic phenomena, such as disturbance propagation and creation of a shockwave on freeways and can be easily adapted to account for traffic signal control and ramp metering devices (Ziliaskopoulos, 2000).

More specifically, Daganzo (1994, 1995) showed that if the relationship between traffic flow \( q \) and density \( k \) follows the function form of Equation (1) as depicted in Fig. 3(a) where \( v, q_{\max}, w, k \) and \( k_j \) denote the free-flow speed, maximum flow (capacity), the speed with which disturbances propagate backwards when traffic is congested.

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Fig. 3. (a) The equation of state of the CTM; and (b) a depiction of the various cell types utilized in the CTM model.
(the backward wave speed), density, and the maximum (or jam) density. The LWR equations for a single highway link can be approximated by a set of difference equations with the current conditions (the state of the system) being updated at every time interval:

\[ q = \min\{v_k, q_{\text{max}}, w(k_j - k)\}, \quad \text{for} \quad 0 \leq k \leq k_j. \quad (1) \]

The model creates discrete time periods of interest (assignment periods) which are the small updating intervals (time interval) of the traffic state. Based on the time periods, it divides every link of the street network into small homogeneous segments called cells. The cell size is such that a vehicle moves between adjacent cells within the time interval when traveling at the free-flow speed. The cells are generally classified into ordinary, diverging, merging, source and super-safe cells as depicted in Fig. 3(b). Different types of cells have different associated difference equations to ensure that the traffic dynamics are reasonably represented. For details of the CTM, please refer to Daganzo (1994, 1995).

### 3.2. The JEDFRFD schedule optimization formulation

The JEDFRFD formulation is based on the proposed network transformation and demand specification techniques discussed previously. The network transformation includes the creation of source nodes, super-safe nodes, a hot zone and connectors. The demand specification includes preloading all the evacuation flows into the source node at the beginning of the evacuation. Once the network and demand are both ready, the node-arc network is converted into a CTM-based network. Cells are connected via cell connectors. All the cells can be physically regarded as being ordinary cells, merging cells, or diverging cells. From a demand specification standpoint, all the cells in the hot zone can be source cells but only one sink cell representing the super-safe node should exist. The resulting cell network is a single-sink network. All the evacuation flows emanating from source cells will be assigned and routed through the network to reach the sink cell by solving the JEDFRFD model.

The JEDFRFD formulation is now presented. The used notation generally follows that used in Ziliaskopoulos (2000).

- **C** = set of cells: ordinary (C_O), diverging (C_D), merging (C_M), source (C_S), sink (C_S), destination (C_p = \{i|j|∈Γ^{-1}(k ∈ C_S)\});
- **Ξ** = set of discrete time intervals;
- **x_i^t** = number of vehicles in cell i at time interval t;
- **N_i^t** = maximum number of vehicles that can be accommodated in cell i at time interval t;
- **y_{ij}^t** = number of vehicles moving from cell i to cell j at time interval t;
- **h** = set of cell connectors: ordinary (h_O), merging (h_M), diverging (h_D), source (h_R), and sink (h_S);
- **Q_i^t** = maximum number of vehicles that can flow into or out of cell i during time interval t;
- **v** = link free-flow speed;
- **w** = backward propagation speed;
- **δ_i^t** = ratio v/w for each cell i and time interval t;
- **Γ(i)** = set of successor cells to cell i;
- **Γ^{-1}(i)** = set of predecessor cells to cell i;
- **d_i^t** = demand (inflow) at cell i at time interval t;
- **t** = total number of evacuees to be evacuated at cell i;
- **x_i** = flow at cell i at the beginning of evacuation.

It is noted that the proposed network transformation determines the definition of the set C, and its subsets. Set C includes only those cells in the hot zone plus the newly created sink cell. Each source node in the node-arc network discussed previously corresponds to one source cell in the CTM network. All the source cells are included in the set C_R. The super-sink node in the node-arc network corresponds to one sink cell and C_S contains only this sink cell:

\[
\min \sum_{\forall i \in C} \sum_{\forall t \in \mathcal{I}} x_i^t, \quad (2)
\]

subject to

\[
x_i^t - x_i^{t-1} - \sum_{k \in \Gamma^{-1}(i)} y_{ki}^{t-1} + \sum_{j \in \Gamma(i)} y_{ij}^{t-1} = 0, \quad \forall i \in C \setminus \{C_R, C_S\}, \forall t \in \mathcal{I}, \quad (3)
\]

\[
y_{ij}^t - y_{ij}^{t-1} \leq 0, \quad y_{ij}^t \leq Q_i^t, \quad y_{ij}^t + \delta_i^t x_i^t \leq \delta_j^t N_j^t, \quad \forall (i, j) \in h_O \cup h_R, \forall t \in \mathcal{I}, \quad (4)
\]

\[
y_{ij}^t - y_{ij}^{t-1} \leq 0, \quad y_{ij}^t \leq Q_i^t, \quad \forall (i, j) \in h_S, \forall t \in \mathcal{I}, \quad (5)
\]

\[
y_{ij}^t \leq Q_i^t, \quad y_{ij}^t + \delta_i^t x_i^t \leq \delta_j^t N_j^t, \quad \forall (i, j) \in h_D, \forall t \in \mathcal{I}, \quad (6)
\]

\[
\sum_{i \in \Gamma(i)} y_{ij}^{t-1} - \sum_{j \in \Gamma(i)} y_{ij}^t = 0, \quad \forall i \in C_D, \forall t \in \mathcal{I}, \quad (7)
\]

\[
\sum_{i \in \Gamma^{-1}(j)} y_{ij}^t - \sum_{i \in \Gamma^{-1}(j)} y_{ij}^{t-1} \leq \delta_j^t N_j^t, \quad \forall (i, j) \in h_M, \forall t \in \mathcal{I}, \quad (8)
\]

\[
x_i^t - x_i^{t-1} + y_{ij}^t - y_{ij}^{t-1} = d_i^t, \quad \forall j \in \Gamma(i), \forall i \in C_R, \forall t \in \mathcal{I}, \quad (9)
\]

\[
d_i^{t-1} = \begin{cases} \hat{d}_i, & \forall i \in C_R, \forall t = 1, \\ 0, & \forall i \in C_R, \forall t > 1, \end{cases} \quad (10)
\]

\[
y_{ij}^t = 0, \quad \forall (i, j) \in h, \forall t \in \mathcal{I}, \quad (12)
\]

\[
x_i^t = \delta_i, \quad \forall i \in C, \forall t = 0, \quad (13)
\]

\[
y_{ij}^t \geq 0, \quad \forall i \in C, \forall t \in \mathcal{I}, \quad (14)
\]

\[
x_i^t \geq 0, \quad \forall (i, j) \in h, \forall t \in \mathcal{I}. \quad (15)
\]

Equation (2) is the objective function which aims to minimize the total system travel time for all cells (excluding the sink cell) over the entire planning horizon Ξ, that is \(\sum_{\forall t \in \mathcal{I}} \sum_{\forall i \in C \setminus C_S} x_i^t\). Since the time increment of τ is assumed to be one time unit, τ is removed from the
formulation. Constraint (3) stands for the flow conservation at each cell excluding the source and sink cells. Constraints (4) to (9) correspond to the CTM cell outflow constraints stated in Equation (1). Constraint (10) defines the flow conservation and the initial condition at the source cells. Constraint (11) correspond to the previously discussed demand specification in that the total number of evacuees for cell \(i\) and \(t_i\) are loaded into the source cell only at time 0. Constraint (12) stand for flows at connectors that are initialized to zero. Constraint (13) indicates that at beginning of evacuation, the network is loaded with pre-existing traffic flows \(\hat{x}_i\). Constraints (14) and (15) specify that the cell and connector flows are non-negative throughout the evacuation operation period.

The optimal solution of the JEDRFD formulation given by Equations (2)–(15) characterizes the joint destination-route flow-departure schedule decision. At optimality, the time-dependent flow rate \(y_{ij}^t\) of the inbound connectors \(j \in \Gamma^{-1}(i)\) of each destination cell \(i \in C_p\) determine the time-dependent arrival of evacuees at each destination cell. A destination cell is not considered active if it does not receive any evacuation flow over the entire evacuation period. The optimal traffic assignment is represented by \(\hat{x}_i\) and the flow rates of the cell itself inbound \(y_{ij}^t, j \in \Gamma^{-1}(i)\) and outbound connectors \(y_{ij}^t, j \in \Gamma(i)\). The optimal time-dependent flow rate for the connector from the source cell to the origin cell \(y_{ij}^t, \forall i \in C_R, j \in \Gamma(i)\) represents the optimal discharge of evacuation flow from the source cell into the network. This characterizes the optimal evacuation departure schedule at each evacuation origin.

The JEDFRD model is seemingly similar to the model proposed by Ziliaskopoulos (2000) in that both are single-sink SO DTA models based on CTM with the objective of minimizing the total system travel time. The original formulation was proposed for regular traffic operation. However, this SO formulation has no practical meaning in regular transportation planning or operation. Instead, the all-origin-to-all-destination traffic assignment under the User Equilibrium (UE) principle is of primary interest from a traffic management standpoint. A UE formation of the CTM model has been developed by Ukkuusuri (2002), Ukkuusuri et al. (2004), Ukkuusuri and Waller (2004), Karoonsoontawong and Waller (2005), Ukkuusuri and Waller (2005), and Waller and Ukkuusuri (under review). The JEDFRD formulation is conceived through a series of network transportation and demand specification approaches specifically tailored for no-notice evacuation. The basic concept and motivation is completely different although the final formulation appears to be rather comparable.

The JEDRFD formulation is advantageous in modeling no-notice mass evacuations because its optimal solution encompasses all multidimensional decisions needed for evacuation operation. This LP formulation, however, does not prevent vehicle holding at cells (for details see Ziliaskopoulos (2000)), which is cumbersome for regular traffic operation, but has a particular meaning in the context of evacuation. Vehicle holding may be interpreted as the utilization of control measures to regulate flow on particular roadways and/or intersections by emergency management officers in order to implement the evacuation solution. By limiting access at strategic locations, it may be feasible to move the system’s true performance towards the solved objective function. In the next section, we illustrate the complete modeling procedure through a numerical example.

4. Numerical example

This section provides an illustrative example to highlight the modeling techniques discussed in the preceding sections. The test network, inspired by Ziliaskopoulos (2000), is an eight-node network as illustrated in Fig. 4(a). The evacuation is assumed to proceed east-bound from the west side of the network. The first step of the modeling process is to define the hot zone, warm zone, safe zone perimeters and the operational horizon. Thus, nodes A, D, and G are defined as the evacuation source nodes at which the total number of evacuees are assumed known. Evacuees are considered safe when they reach node C, F, or H. These three safe nodes are further connected to a virtual safe sink node S via three virtual connectors. The modeling of the evacuation becomes equivalent to sending all flow to the virtual sink node in this transformed single-destination network, see Fig. 4(b). The network topology characteristics are summarized in Table 1. All the links are 500 feet in length, except link BC which is 1000 feet in length. All links have two lanes, with a total maximum flow equal to 4320 vehicle per hour (vph).

The next step is to create the equivalent cell network. The size of the time interval for updating the traffic state is assumed to be 10 seconds. The disaster is assumed to occur at time 0, which requires the no-notice evacuation to begin at time 1. The operational horizon of the evacuation is assumed to extend from time 1 to time 10. Given that the speed limit is 50 feet/second, the cell length should be defined as 500 feet so that the assumption pertaining to the CTM is maintained (all vehicles move from one cell to the immediately downstream cell over the 10 second time interval). Figure 4(c) depicts the equivalent cell network of the example network consisting of 14 cells, with cells 1, 5, and 9 being the source cells and cell 14 being the virtual sink cell. It is further specified that cells 1, 5, 9, and 14 have an infinite (or sufficiently large) capacity. Cells 2 to 13 are general cells including both nodes and links. More specifically, cell 2 consists of node A and link AB, cell 6 includes node D and link AD, cell 10 includes node G and link DG, cell 7 is composed of node E and link DE, cell 12 represents link EC, and cell 13 represents link EH. The super-safe node is directly converted to a cell and is assumed
to have an infinite capacity so that it can accommodate all inbound evacuation flows.

It should be noted that for no-notice evacuation modeling, all the evacuation flows are considered to be loaded into the source cells at time 0 which allows the LP model to determine the time-dependent discharge out of the source cells. This demand loading requirement is distinctly different from what is usually considered in the short-notice (e.g., hurricane) evacuation scenario, in which evacuation participation and departure times are estimated or predicted (unless phased evacuation is of concern) using certain econometric approaches (Fu and Wilmot, 2003; Wilmot and Mei, 2003). In the no-notice evacuation case, everybody is required to be evacuated and they should all be ready to be immediately evacuated at the beginning of the evacuation. This is the reason why flows are loaded into the source cells at time 0.

The time-invariant cell properties are listed in Table 2. All the ordinary cells have a maximum flow rate $Q_i$ of 12 vehicles per time interval (10 seconds) except cell 3 that has a temporary capacity reduction from time 1 to time 5 (see Table 3) which simulates a preschedule capacity reduction event during the evacuation. The capacity value ($N_i$) for each cell is determined by the length of the cell and the number of lanes.

A total of 310 decision variables and 540 functional constraints are created for this problem. The optimal route-flow solution, as listed in Table 4, indicates that at the end of time 10, all 74 evacuation flow units reach the safe zone with an optimal total system travel time of 4140 seconds. The solution shows that at time 1, all flows are loaded into source cells 1, 5, and 9. Twelve units of cell 1 flows are discharged to cell 2 at time 2, and the same amount of flow is assigned to cell 6 from cell 5. Another 12 units of flow are assigned to

### Table 1. Geometric characteristics of the example network

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>AD</th>
<th>BC</th>
<th>EC</th>
<th>DE</th>
<th>DG</th>
<th>GH</th>
<th>EH</th>
<th>EF</th>
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<td>1000</td>
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<td>Speed limit (feet/second)</td>
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</tr>
<tr>
<td>Max flow (vph)</td>
<td>4320</td>
<td>4320</td>
<td>4320</td>
<td>4320</td>
<td>4320</td>
<td>4320</td>
<td>4320</td>
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</tbody>
</table>
If an incident is missed in one particular stage, it can always be incorporated. This updated information can include any available data that are related to the present research. Most existing DTA approaches (whether analytical or simulation-based) are generally not sufficiently computationally efficient to be applied to real-time operation in a large metropolitan network. These models generally involve the calculation of many-to-many time-dependent shortest paths, repeated and iterative decomposition or diagonalization of huge matrices, or simulation of million of vehicles’ with time-dependent trajectories, etc. (for an overview of DTA models, please see Mahmassani (2001) and Peeta and Ziliaskopoulos (2001)). It should be noted that following the proposed network transformation approach, the JEDRFD model is a single-destination structure. This represents a great computational advantage particularly if the proposed network transformation approach is integrated with the type of SO DTA model that solves Time-Dependent Shortest Path (TDSP) problems (such as DYNASMART-P (Mahmassani, 2001)) the computational dimension reduces from many-to-many to many-to-one. This saving is significant as the TDSP is usually the most time-consuming step in the SO DTA solution procedure.

While the no-notice evacuation modeling concept and technique are the focal point of this paper, it is important to discuss real-time operation issues such as computation and data that are related to the present research.

More importantly, the model solution indicates how much flow should be assigned to each destination. The flows assigned though the destination cells 4, 8, and 11 can be found by checking the outflow variables $y_{s,t}^{4,14}$, $y_{s,t}^{8,14}$, and $y_{s,t}^{11,14}$ associated with connectors (4, 14), (8, 14) and (11, 14) as listed in Table 4. There are a total of 12 units of flow that are evacuated via cell 4 between times 7 and 8, while 14 units go through cell 8 at times 4, 5, 7, and 8. A total of 48 units go through cell 11 from times 3 to 9. One can also observe that the number of vehicles present at cell 3 is limited to two units of flow going though from times 3 to 6. Note that a dynamic capacity reduction is defined in cell 3 that blocks flow from times 1 to 5. The route-flow solution verifies the capability of the model to respond to the dynamic network topology variation. While it is generally difficult to incorporate unexpected capacity reduction (e.g., incident) when computing the solution for the JEDRFD model is cyclic at the frequency of the roll period.

While the no-notice evacuation modeling concept and technique are the focal point of this paper, it is important to discuss real-time operation issues such as computation and data that are related to the present research. Most existing DTA approaches (whether analytical or simulation-based) are generally not sufficiently computationally efficient to be applied to real-time operation in a large metropolitan network. These models generally involve the calculation of many-to-many time-dependent shortest paths, repeated and iterative decomposition or diagonalization of huge matrices, or simulation of million of vehicles’ with time-dependent trajectories, etc. (for an overview of DTA models, please see Mahmassani (2001) and Peeta and Ziliaskopoulos (2001)). It should be noted that following the proposed network transformation approach, the JEDRFD model is a single-destination structure. This represents a great computational advantage particularly if the proposed network transformation approach is integrated with the type of SO DTA model that solves Time-Dependent Shortest Path (TDSP) problems (such as DYNASMART-P (Mahmassani, 2001)) the computational dimension reduces from many-to-many to many-to-one. This saving is significant as the TDSP is usually the most time-consuming step in the SO DTA solution procedure.

These models usually require evacuation travel demand (departure time, origins and destinations, etc.) to be known a priori and be taken as model input to solve for the traffic flow assignment based on the given demand information. For a typical urban transportation application,

### Table 2. Time invariant cell properties

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<tr>
<th>Cell</th>
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<th>3</th>
<th>4</th>
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<th>6</th>
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<tr>
<td>$Q_t$</td>
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<td>12</td>
<td>* 12</td>
<td>12</td>
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*See Table 3 for time-dependent data.

### Table 3. Time-dependent cell properties

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### Table 4. Optimal solution for minimal exposure objective

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<td>36</td>
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<td>60</td>
<td>72</td>
<td>74</td>
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</table>

Optimal Solutions $y_{s,t}^j$ for sink node inbound connectors

| $y_{s,t}^4,14$ | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 2   | 0   | 0  |
| $y_{s,t}^5,14$ | 0   | 0   | 0   | 0   | 4   | 4   | 0   | 4   | 2   | 0   | 0  |
| $y_{s,t}^{11,14}$ | 0   | 0   | 0   | 12  | 8   | 8   | 12  | 6   | 0   | 2   | 0  |
this demand information is usually obtained from other external travel demand models that typically follow the procedure of trip generation and trip distribution based on trip diary surveys regularly conducted by transportation agencies. For an evacuation operation purpose, the evacuation demand can be addressed from both descriptive and prescriptive perspectives. The descriptive perspective focuses on predicting how evacuees may choose their evacuation destinations, routes and departure time. Traffic control strategies are devised based on this demand. Accurately predicting evacuation demand during an evacuation is a very complex and possibly intractable task because of the numerous uncertainties that can occur during the crisis. No reliable and accurate approach to predicting the evacuation demand currently exists in the literature.

For evacuation purposes, proactively managing demand and implementing the corresponding traffic control strategies is a more meaningful operational concept than passively predicting and reacting to demand because the performance degradation of a transportation network non-linearly increases with the level of network traffic loading. Undoubtedly, controlling evacuation demand is a significant practical challenge; understanding how the extent of noncompliance affects the model effectiveness also requires further research.

For real-time operation purposes, the JEDRFD model can be implemented in a Rolling Horizon (RH) fashion. Doing so reduces the required time domain as well as model dimension and computation time. The RH strategy also allows for frequent cyclic updates of model inputs and parameters according to unfolding traffic conditions which may be subject to a vast amount of uncertainty. The RH implementation allows the JEDRFD model to be regularly reoptimized to ensure the model solutions remain realistic throughout the operation horizon.

Data needed by the proposed model primarily fall into two categories: (i) the total number of evacuees; and (ii) the prevailing traffic conditions. The former deals with how many people are present and how they are distributed in the hot zone at time of evacuation. This can be estimated using demographic and travel activity information. Most metropolitan planning organizations in the US metropolitan regions maintain and update such information.

Real-time traffic data are needed to estimate the network initial flow condition. This used to be a major issue for evacuation operation in the last two decades, but this becomes less of an issue after decades of ITS deployment in most major US cities. Nowadays, sensor data from a variety of sources (stationary loops underneath highway traffic lane pavements, automatic vehicle identification from toll roads, GPS from truck fleets, even cellular phone data from wireless carriers, etc.) have been compiled and integrated into traffic management centers that can be directly fed into the model.

5. Concluding remarks

In this paper we present a network transformation, demand specification and a SO DTA modeling technique to solve the JEDRFD optimization problem for no-notice mass evacuations. The main contribution of this paper is that it proposes a network transformation and demand modeling technique that allows the optimal evacuation destination, traffic assignment and evacuation departure schedule decisions to be formulated into a unified optimal traffic flow optimization model by solving these decisions simultaneously. This is the first time that such a unified modeling approach has been proposed. Another contribution is that the proposed modeling procedure can be integrated with either simulation-based or analytical DTA frameworks, which means that the presented research is applicable to a wide range of models and tools familiarized by researchers in both transportation and industrial engineering areas.

For a real-time application, the proposed approach is capable of further considering unfolding roadway conditions such as reduced roadway capacities due to earthquake damage, road blocks, and unserviceable roads. Evacuating vehicles can be reassigned to new safe zones in order to continuously reoptimize the evacuation operation.

Future research could include testing the proposed method with different evacuation objectives on an actual metropolitan network, as well as the extension to the short-notice evacuation situation. Real-time operational issues such as the RH-based reoptimization implementation and the update of network initial conditions at each reoptimization are also currently being studied.

Acknowledgements

This research was partially supported by the National Science Foundation through grant SES-0332001. The authors assume the responsibility for the facts presented, viewpoints expressed, and accuracy of the data in this paper.

References


Biographies

**Yi-Chang Chiu** is an Assistant Professor in the Department of Civil Engineering and Engineering Mechanics at the University of Arizona. His research interests include the theoretical development and application of dynamic traffic assignments, large-scale vehicular traffic evacuation modeling, system dynamics and interdependence modeling, and intelligent transportation systems modeling including dynamic message signs, location configuration and freeway travel time prediction modeling and wireless sensor configuration design. He received a Ph.D. degree in Transportation Engineering from the University of Texas at Austin. Prior to joining the University of Arizona, he was an Assistant Professor at the University of Texas at El Paso, a research engineer at the Center for Transportation Research (CTR) at UT-Austin, and a senior technical staff member at Nortel Networks Inc.

Hong Zheng is a doctoral student at the University of Arizona under the guidance of Yi-Chang Chiu. His major research interests are in large-scale evacuations and traffic emergency response as well as transportation planning and network flow modeling. Prior to his studies at the University of Arizona, he received his M.S.C.E. at the Beijing University of Technology, China and worked for the China Highway Engineering Consulting and Supervision Corporation.

**Jorge Villalobos** is both a Research Engineer and a Ph.D. student in the Civil Engineering Department at the University of Arizona; collaboratively working with Yi-Chang Chiu. He was recently awarded a National Science Foundation Graduate Fellowship. He is currently working towards developing a large-scale network mesoscopic simulation and dynamic traffic assignment model utilizing distributed computing systems. Prior to joining the University of Arizona to obtain his Ph.D., he worked for Shell Oil Products US as a project engineer for off-shore pipeline construction, an operations engineer for the on-shore pipeline and product distribution terminal supervisor. His interest in modeling aging transportation infrastructure, system dynamics, advanced decision making, and risk mitigation led him to undertake the Ph.D.

Bikash Gautam is a M.S. student at the University of Texas at El Paso (UTEP) under the guidance of Yi-Chang Chiu. He is developing his interests in intelligent transportation systems, traffic operations, transportation network optimization and evacuation modeling, planning and analysis as part of his studies. Prior to his studies at UTEP, he received his B.E. in Civil Engineering at the Tribhuvan University, Institute of Engineering, Pulchowk in Kathmandu, Nepal.