An Initial Acquisition Scheme for Walsh-Hadamard Code Division Multiplexing

Toshiharu Kojima and Yuki Nagashima
Graduate School of Informatics and Engineering
The University of Electro-Communications
Chofu, Tokyo, 182-8585, Japan
Email: {Kojima.Toshiharu@, n1631109@edu.cc.}uec.ac.jp

***Abstract*—This paper addresses the problem of initial acquisition in code division multiplexing employing Walsh-Hadamard code (WHCDM). Unlike pseudo-noise codes used in direct-sequence spread spectrum communications, Walsh-Hadamard code has many side lobes in its autocorrelation function. Well-known initial acquisition schemes for direct-sequence spread spectrum communications, therefore, cannot be applied to WHCDM. In order to solve this problem, we propose an initial acquisition scheme for WHCDM with a novel approach. It detects the symbol timing on the basis of not the autocorrelation function of Walsh-Hadamard code but the energy variance of the signal after demultiplexing. The results of computer simulation confirm that the proposed scheme provides rapid acquisition.**

I. Introduction

Walsh-Hadamard (WH) code is an orthogonal code consisting of only $+1$ and -1. This property makes WH code suitable for code division multiplexing (CDM) [1]. In CDM employing WH code (WHCDM), the multiplexing and the demultiplexing are easily implemented by WH transform (WHT).

WHCDM is attractive for helicopter satellite communications. In helicopter satellite communications [2], the transmitted signal is periodically blocked by the rotor blades. This blockage significantly degrades the bit error rate (BER) performance. We showed that WHCDM has an implicit time diversity effect in order to overcome the periodic blockage and improves the BER performance at high transmission speed [3].

CDM usually requires symbol (or code) synchronization in the receiver. The symbol synchronization consists of two steps, namely, initial acquisition and tracking [4]. This paper addresses the initial acquisition in WHCDM. In direct-sequence spread spectrum (DSSS) communications, employing a pseudo-noise (PN) code as the spreading code, the initial acquisition is performed on the basis of the impulse-like autocorrelation function of the PN code [5]. However, there are a lot of sidelobes in the autocorrelation function of WH code. In consequence, the initial acquisition scheme for DSSS communications is not applicable to WHCDM. There exists, to the best of the our knowledge, no initial acquisition scheme for WHCDM.

In this paper, we propose an initial acquisition scheme for WHCDM using M-ary phase-shift keying (MPSK) as the primary modulation. The proposed scheme acquires the symbol timing on the basis of the energy variance of the demultiplexed signal. The results of computer simulation show that the proposed scheme achieves rapid acquisition even under the periodic blockage environment in helicopter satellite communications.

The rest of the paper is organized as follows: Section II illustrates the communication system model. In Section III, the principle and the configuration of the proposed initial acquisition scheme are described. We evaluate the performance of the proposed scheme by computer simulation in Section IV. Finally, the paper is concluded in Section V.

II. Communication System Model

The communication system model is illustrated in Fig. 1. In this model, binary phase-shift keying (BPSK) is employed as the primary modulation of WHCDM for simplicity. Let N denote the multiplexing factor of WHCDM. It is equal to the order of WHT performed in the transmitter and the receiver.

A. Transmitter

In the transmitter, the binary information data $a_i \in \{0, 1\}$ is supplied to the BPSK modulator and the primary modulated signal b_i is produced. Letting E_b denote the signal energy per bit, the primary modulated signal b_i is represented as

$$b_i = \sqrt{E_b} \cos a_i \pi. \tag{1}$$

In the serial-to-parallel converter, the following primary modulated symbol $\boldsymbol{B}_n$ of length N is formed:

$$\boldsymbol{B}_n = \left[b_{(n-1)N}, b_{(n-1)N+1}, \cdots, b_{nN-1} \right]$$

The WHT processor transforms the primary modulated symbol $\boldsymbol{B}_n$ into the transmitted CDM symbol $\boldsymbol{S}_n$ given by

$$\boldsymbol{S}_n = \frac{1}{\sqrt{N}} \boldsymbol{B}_n \boldsymbol{H} \tag{2}$$

where $\boldsymbol{H}$ is Walsh-Hadamard matrix of order N. Each element of the transmitted CDM symbol $\boldsymbol{S}_n$ is consecutively output from the parallel-to-serial converter, resulting in the discrete transmitted signal. Let the i-th element of the transmitted CDM symbol $\boldsymbol{S}_n$ be represented by $s_{(n-1)N+i}$.

B. Channel

In the transmission channel, the continuous transmitted signal $s(t)$ is expressed as

$$s(t) = \sum_{i=-\infty}^{\infty} s_i c(t - iT_c) \tag{3}$$

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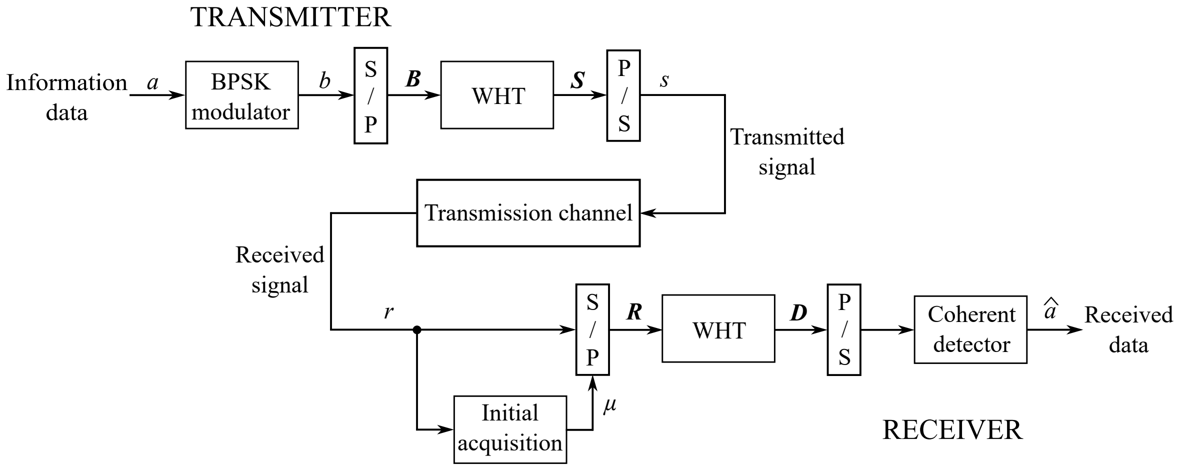

Fig. 1. Communication system model.

where

$$c(t) = \begin{cases} 1 & (0 \leq t < T_c) \\ 0 & \text{(otherwise)} \end{cases}. \tag{4}$$

and T_c denotes the chip duration. The transmitted signal $s(t)$ is periodically blocked by the rotor blades of the helicopter. Letting T_p and T_d represent the blockage period and duration, respectively, the periodic blockage is described by the following function [3]:

$$h(t) = \sum_{k=-\infty}^{\infty} u(t - kT_p) \tag{5}$$

where

$$u(t) = \begin{cases} 1 & (T_d \leq t < T_p) \\ 0 & \text{(otherwise)} \end{cases}. \tag{6}$$

The continuous received signal $r(t)$ is given by

$$r(t) = h(t)s(t) + n(t) \tag{7}$$

where $n(t)$ is additive white Gaussian noise (AWGN) with the single-sided power spectral density N_0.

C. Receiver

In the receiver, the discrete received signal $r_i = r(iT_c)$ is first input to the initial acquisition scheme and the symbol timing of the received signal is detected. On the basis of the detected symbol timing $\mu \in \{-N/2, \cdots, 0, \cdots, N/2-1\}$, the serial-to-parallel converter forms the following received CDM symbol $\boldsymbol{R}_n$:

$$\boldsymbol{R}_n = \left[r_{(n-1)N+\mu}, r_{(n-1)N+\mu+1}, \cdots, r_{nN+\mu-1} \right]$$

The received CDM symbol $\boldsymbol{R}_n$ is transformed into the demultiplexed symbol $\boldsymbol{D}_n$ in the WHT processor. The demultiplexed symbol $\boldsymbol{D}_n$ is represented as follows:

$$\boldsymbol{D}_n = \frac{1}{\sqrt{N}} \boldsymbol{R}_n \boldsymbol{H}. \tag{8}$$

After parallel-to-serial conversion, each element of the demultiplexed symbol $\boldsymbol{D}_n$ is supplied to the coherent detector and the received data $\hat{a}_i \in \{0, 1\}$ is finally obtained.

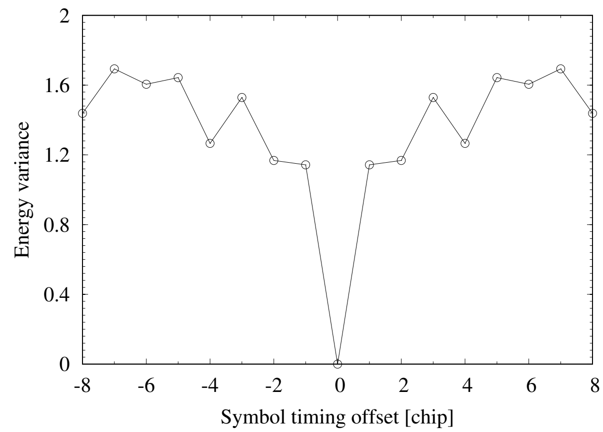

Fig. 2. Dependence of the energy variance on the symbol timing offset.

III. Proposed Scheme

We pay attention to the following fact: All elements of the primary modulated symbol $\boldsymbol{B}_n$ in the transmitter have the same energy. That is,

$$\left| b_{(n-1)N} \right|^2 = \left| b_{(n-1)N+1} \right|^2 = \cdots = \left| b_{nN-1} \right|^2 = E_b \tag{9}$$

In the case of $r(t) = s(t)$ (i.e. noiseless channel), the received CDM symbol $\boldsymbol{R}_n$ is identical with the transmitted CDM symbol $\boldsymbol{S}_n$ when the detected symbol timing μ is correct. Then, the demultiplexed symbol $\boldsymbol{D}_n$ has the same elements as the primary modulated symbol $\boldsymbol{B}_n$ and therefore all elements of $\boldsymbol{D}_n$ have the same energy. In other words, when the detected symbol timing is correct, the value of the energy variance of the demultiplexed symbol $\boldsymbol{D}_n$ is zero. On the other hand, it takes a positive value when the detected symbol timing is offset from the correct timing.

Fig. 2 shows the numerical results for the dependence of the energy variance on the symbol timing offset under the condition of $N = 16$ and $E_b = 1$. It can be seen that the energy variance has a negative peak at the correct symbol timing. In addition, there are no sidelobes. The energy variance of the signal after demultiplexing is thus impulse-like, similar to the autocorrelation function of a PN code. We can therefore detect

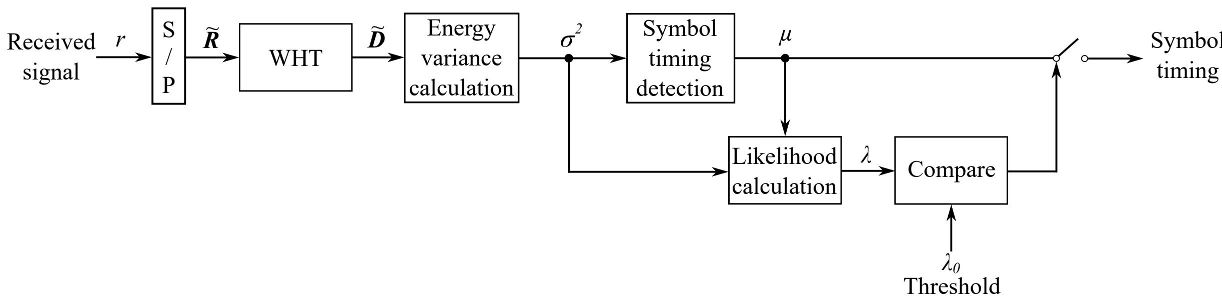

Fig. 3. Configuration of the proposed initial acquisition scheme.

the symbol timing on the basis of the energy variance.

The configuration of the proposed initial acquisition scheme for WHCDM is illustrated in Fig. 3. In the proposed scheme, the serial-to-parallel converter forms the received signal into the following temporary CDM symbols $\tilde{\boldsymbol{R}}_{nm}$ chip by chip:

$$\tilde{\boldsymbol{R}}_{nm} = \left[r_{(n-1)N+m}, r_{(n-1)N+m+1}, \cdots, r_{nN+m-1}\right]$$
$$m = -N/2, \cdots, 0, \cdots, N/2-1$$

These temporary CDM symbols $\tilde{\boldsymbol{R}}_{nm}$ are transformed into the temporary demultiplexed symbols $\tilde{\boldsymbol{D}}_{nm}$ by WHT processor. That is,

$$\tilde{\boldsymbol{D}}_{nm} = \frac{1}{\sqrt{N}} \tilde{\boldsymbol{R}}_{nm} \boldsymbol{H}. \tag{10}$$

Next, the energy variance $\sigma_n^2(m)$ of each temporary demultiplexed symbol $\tilde{\boldsymbol{D}}_{nm}$ is calculated. Letting d_{nmi} denote the i-th element of $\tilde{\boldsymbol{D}}_{nm}$, the energy variance $\sigma_n^2(m)$ is given by

$$\sigma_n^2(m) = \frac{1}{N} \sum_{i=0}^{N-1} |d_{nmi}|^4 - \left(\frac{1}{N} \sum_{i=0}^{N-1} |d_{nmi}|^2 \right)^2 \tag{11}$$

The symbol timing μ is decided as follows:

$$\mu = \arg\min_{m} \sigma_n^2(m) \tag{12}$$

It is clear that the detection process requires only one symbol duration $T_s = NT_c$.

At the same time, in order to prevent the false alarm, the likelihood λ of the detected symbol timing μ defined as

$$\lambda = \frac{\sigma_n^2(\mu)}{\min\left\{\sigma_n^2(m), m \neq \mu\right\}} \tag{13}$$

is calculated. From Fig. 2, it is apparent that in the noiseless channel, the detected timing μ is correct and the value of the likelihood λ is zero. On the other hand, when the negative peak at the correct symbol timing is lost for some reason, the detected timing is unreliable and the likelihood λ takes a value close to 1. The likelihood λ is thus negatively correlated with the reliability of the detected symbol timing. Accordingly, when the likelihood λ is greater than a predetermined threshold λ_0, the detected symbol timing μ is rejected (i.e. missed detection) and the detection process is repeated.

Note that all elements of the primary modulated symbol $\boldsymbol{B}_n$ have the same energy and the value of its energy variance is zero not only in BPSK but also in MPSK. In consequence, we can also apply the proposed scheme to WHCDM using MPSK as the primary modulation.

TABLE I. SIMULATION CONDITIONS.

Primary modulation	BPSK		
Transmission data rate	64 kbps	256 kbps	1 Mbps
Multiplexing factor N	1024	4096	16384
Threshold λ_0	0.95	0.98	0.99
Blockage period T_p	38.9 msec		
Blockage duration T_d	3.3 msec		
Blockage ratio ρ	0.086		

IV. COMPUTER SIMULATION

We evaluate the performance of the proposed initial acquisition scheme in AWGN channel and the periodic blockage channel by computer simulation. Table I summarizes the simulation conditions. The chip rate of WHCDM signal is equal to the transmission data rate because the primary modulation is BPSK. The multiplexing factor N at each data transmission rate is selected to maximize the implicit time diversity effect of WHCDM in the periodic blockage channel. The blockage ratio ρ is defined as [3]

$$\rho = \frac{T_d}{T_p}$$

and $\rho = 0.086$ is a typical value. As mentioned in Section I, to the best of the our knowledge, there is no conventional scheme to be compared with the proposed scheme. From the viewpoint of the practical use in helicopter satellite communications, we set the target for the acquisition probability to be at least 0.95 at E_b/N_0 = 0 dB in the periodic blockage channel.

A. Acquisition performance in AWGN channel

Fig. 4 shows the acquisition performance of the proposed scheme in AWGN channel. We can see that the acquisition performance is enhanced with increasing the multiplexing factor N. This is because the energy per symbol E_s of WHCDM signal is directly proportional to the multiplexing factor (i.e. $E_s = NE_b$). It is clear that the acquisition probability for each N is greater than 0.95 at E_b/N_0 = 0 dB. These results indicate that the proposed scheme provides the excellent acquisition performance in AWGN channel.

Furthermore, we confirm that the probability of the false alarm is negligibly small. Specifically, it is less than 10^{-3} when the acquisition probability is at least 0.95 for all N. This shows that the rejection of the detected symbol timing μ based on the comparison between the likelihood λ and the appropriate threshold λ_0 is accurate and useful.

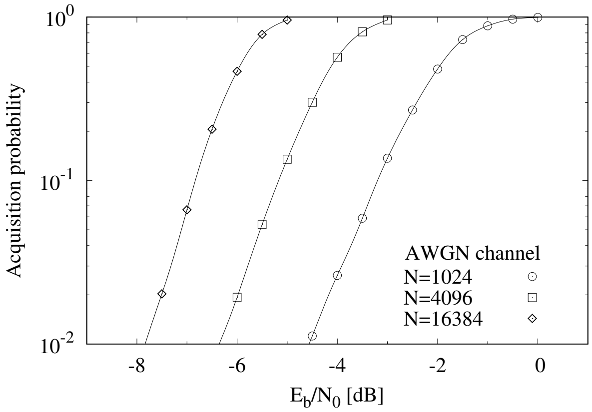

Fig. 4. Acquisition performance in AWGN channel.

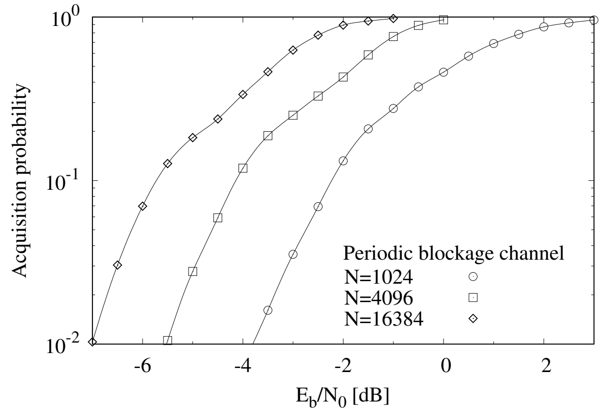

Fig. 5. Acquisition performance in the periodic blockage channel.

B. Acquisition performance in the periodic blockage channel

The acquisition performance of the proposed scheme in the periodic blockage channel is presented in Fig. 5. It can be seen that in comparison with Fig. 4, the acquisition performance in Fig. 5 is severely degraded by the periodic blockage. However, in the case of N = 4096 and 16384, the acquisition probability exceeds the target value 0.95 at E_b/N_0 = 0 dB. On the other hand, our target is not achieved when N = 1024.

We introduce the recursive integration [5] into the proposed scheme in order to improve the acquisition performance of N = 1024. The proposed scheme combined with the recursive integration decides the symbol timing μ as follows:

$$\mu = \arg\min_{m} \sum_{k=0}^{K-1} \sigma_{n+k}^2(m) \tag{14}$$

where K is the number of recursions. The processing time of the symbol timing detection is increased up to K times (i.e. KT_s) with the recursive integration. Then, the likelihood λ in order to prevent the false alarm is calculated by the following equation:

$$\lambda = \frac{\displaystyle\sum_{k=0}^{K-1} \sigma_{n+k}^2(\mu)}{\min\left\{\displaystyle\sum_{k=0}^{K-1} \sigma_{n+k}^2(m), m \neq \mu\right\}} \tag{15}$$

Fig. 6 shows the acquisition performance of N = 1024 with the recursive integration in the periodic channel. We can see that the recursive integration with only K = 2 significantly improves the acquisition performance. When K = 3, although the acquisition performance is more improved, the amount of improvement in E_b/N_0 terms is reduced in comparison with K = 2. As a result, the number of recursions K of 2 is sufficient to achieve our target, i.e. the acquisition probability greater than 0.95 at E_b/N_0 = 0 dB. Although the recursive integration increases the processing time of the symbol timing detection as mentioned above, it is no more than two symbol duration $2T_s$ when K = 2. These results confirm that the proposed scheme provides the rapid acquisition even in the periodic blockage channel.

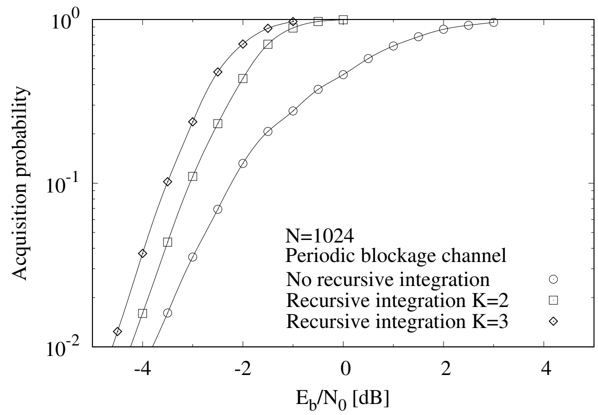

Fig. 6. Acquisition performance with recursive integration in the periodic blockage channel.

V. Conclusion

In this paper, we have proposed an initial acquisition scheme for WHCDM using MPSK as the primary modulation. It detects the symbol timing on the basis of not the autocorrelation function of WH code but the energy variance of the signal after demultiplexing. The results of computer simulation have shown that the proposed scheme achieves rapid acquisition even under the severe environment in helicopter satellite communications. We can therefore conclude that the problem of initial acquisition in WHCDM is solved.

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