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Numerical analysis of an embankment on soft soils considering large displacements

Paulo J. Venda Oliveira, Luís J.L. Lemos [1]

Department of Civil Engineering, University of Coimbra, Rua Luís Reis Santos, 3030-788 Coimbra, Portugal

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ABSTRACT

The behaviour of an embankment built on a Portuguese soft soil is analysed considering the material and geometric non-linearity associated with a coupled soil–water formulation. The numerical predictions are compared with the field data in terms of settlements, horizontal displacements and excess pore water pressures. The repercussions of including the large displacements formulation are also studied. It is found that the analysis considering large displacements results in a decrease in settlements and an increase in the rate of excess pore pressure dissipation, both of which are related to the reduction of the thickness of a deformable layer.

1. Introduction

The technological development that has occurred in recent decades has helped our understanding and forecasting of a wide variety of phenomena, some of which cut across several areas of scientific knowledge. Geotechnical engineering has naturally profited from this technological progress, especially in terms of improving the prediction of the behaviour of geotechnical works. With the aid of automatic calculation software based on the finite element method, the behaviour of a major geotechnical works can be simulated, no matter how complex it is. Irregular boundary conditions, heterogeneities, complex constitutive models, time-dependent phenomena and so forth are all taken into account.

The analysis of most of these problems usually assumes that both displacements and strains are small and then the infinitesimal linear strain approximation can be used. But this hypothesis is not realistic for structures built on very compressible soils since these soils are subject to high displacements; on the other hand, their saturation and the lower permeability involve a coupled soil–water analysis. As a result, several studies have been published on the implementation of coupled consolidation theories with large displacements (geometric non-linearity), considering either linear elastic materials [1–5] or non-linear material behaviour [6–9], but only a few include applications to real situations. This paper thus intends to clarify the influence of geometric non-linearity on the behaviour of an embankment built on soft soils in Portugal [10].

In order to show the significance of geometric non-linearity three numerical cases were compared, one considering large displacement (“*Large Disp.*”) and the others assuming small displacements, one with (“*Small Disp. UpD*”) and one without (“*Small Disp.*”) updating of the nodal coordinates at the end of each loading and/or time step. The numerical results obtained are compared with the field data in terms of settlements, horizontal displacements and excess pore water pressures.

The numerical results presented are based on the following hypotheses: (i) Biot’s consolidation theory, i.e. the coupled formulation of the interstitial fluid flow and the deformation of the solid skeleton; this formulation is based on Belkeziz and Magnan [11] and Babchia and Magnan [12]; (ii) utilisation of the Modified Cam-Clay (MCC) model, to simulate the constitutive behaviour of the soil foundation; (iii) consideration of a Lagrangian formulation to analyse the geometric non-linearity with updating of nodal coordinates at the end of each step of time and/or load; this formulation was described in Zienkiewick [13]; (iv) the updating of nodal coordinates in one of the two infinitesimal deformations analyses is carried out according to geometrical non-linearity analysis, i.e. at the end of each loading and/or time step; once this updating is done the matrices of deformations, stiffness and coupling are recalculated [14]; (v) in order to simulate the real conditions of execution of the embankment, where it is generally necessary to guarantee the top level of the embankment, the nodal coordinates of new embankment elements above the current top surface of the embankment were updated in order to take the previous settlements into account [15]; naturally, this assumption causes an

Corresponding author. Tel.: +351 239797271; fax: +351 239797267.
E-mail addresses: pjvo@dec.uc.pt (P.J.V. Oliveira), llemos@dec.uc.pt (L.J.L. Lemos).
[1] Tel.: +351 239797263; fax: +351 239797267.

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Nomenclature

e	void ratio
$e_{\lambda o}$	void ratio for a virgin compression line at p' = 1 kPa
H_{emb}	embankment height
k	coefficient of permeability
K_0	coefficient of earth pressure at rest
M	slope of the critical state line in p'–q plane
OCR	overconsolidation ratio
u	pore water pressure
WT	water table
γ	unit weight of soil
κ	slope of the swelling line in e–ln p' plane
λ	slope of the virgin compression line in e–ln p' plane
ν	Poisson's ratio

Subscripts

0	initial
y	vertical
x	horizontal

increase in the load applied to the embankment, relative to the initial conditions.

All the numerical analyses used the FEMCONS program, developed in the Universities of Porto and Coimbra, which can perform elastoplastic analyses with coupled consolidation.

2. Description of the embankment characteristics

The site studied is located in Portugal, at 7.775 km on the A14 motorway. The embankment was instrumented with the following equipment (Fig. 1): (i) a sub-vertical inclinometer tube placed on the vertical of the foot of the main embankment's slope, (ii) two settlement plates (T and M) and an electric piezometer (P). Prefabricated vertical drains (PVDs) were installed under the embankment and the additional berm, down to the bottom of the soft deposit.

The construction of the embankment is described by six sublayers with elevations of 1.1, 1.85, 3.45, 4.7, 7.55 and 8.1 m, applied at 0, 80, 240, 290, 385 and 420 days.

The geotechnical characterisation of these soils, carried out by Coelho [16], allowed the foundation soil to be zoned as shown in Table 1, where the physical and mechanical characteristics of the layers are presented. The behaviour of the foundation soil is simulated with the MCC model, while the behaviour of the embankment is predicted by a linear elastic law, with ν' = 0.3 and the deformability modulus varying between 30 MPa (bottom layer) and 2 MPa (top layer), thereby reproducing the reduced containment of the upper layers.

The effect of PVDs has been modelled by increasing the coefficient of permeability of the subsoil in the vertical direction, with good results [17,18]. Another method of modelling PVD in two-dimensional analysis consists of matching the axisymmetric and plain strain conditions, considering of the effects of smear [19–21] and resistance to flow of water [22,23]. Based on Asaoka et al. [17], a value for the equivalent vertical coefficient of permeability of 12 times the vertical coefficient of permeability was achieved for this soil [14]. Table 1 presents the equivalent vertical coefficients of permeability used in the calculations, which state the drainage conditions of the "soil/PVD" system.

The coefficients of permeability also change with the void ratio during the calculation [15], in accordance with Taylor [24]:

$$k = k_0 \times 10^{\frac{e-e_0}{C_k}} \quad (1)$$

where e_0 represents the initial void ratio, k_0 the coefficient of permeability related to e_0, k the corrected coefficient of permeability related to the actual void ratio e, and C_k is a constant equal to $0.5e_0$ [24].

The finite element mesh used for the plane-strain analysis (Fig. 1) is composed of 679 nodal points and 202 eight-noded isoparametric quadrilateral elements. In order to simulate the consolidation process, elements with 20 nodal degrees of freedom are used below the water table, providing quadratic interpolation of displacements and linear interpolation of pore pressures and thereby allowing the calculation of the displacements in eight nodes and the excess pore pressure in four corner nodes.

The boundary conditions of the mesh are such that the right vertical side is restrained from moving in the horizontal direction, while the bottom boundaries are restrained from moving in both directions. The boundary conditions defined between the rock foundation (limestone) and the upper soil layers (soft soil and embankment) reflect the significant weathering of the limestone, especially near the surface, which creates a wrinkled boundary that provides an effective bond between the rock and the soil [10,17]. In terms of hydraulic conditions, only the top boundary, located at the same level of the water table, is permeable. It is assumed that there is no flow of water through the remaining boundaries.

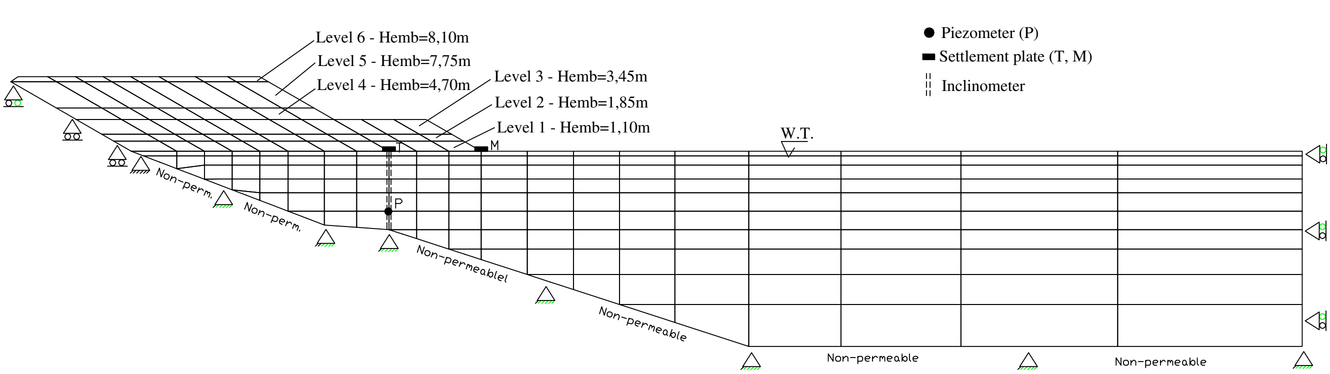


Fig. 1. Finite element mesh.

Table 1
Physical and mechanical characteristics of soil layers [14].

Layer	Depth (m)	γ (kN/m^3)	OCR	K_0	e_o	Parameters of MCC model				$k_{y(eq)}$[a] (m/day) [$\times 10^{-4}$]	k_x/k_y
						$e_{\lambda o}$	λ	κ	M		
1	0.0–0.5	15.0	7.0	0.87	2.0	2.58	0.226	0.028		–	
2	0.5–1.5	15.0	5.0	0.76	2.0	2.76	0.226	0.028		15.6	
3	1.5–3.0	14.8	3.0	0.62	2.1	3.02	0.282	0.035		62.4	
4	3.0–4.5	14.5	1.5	0.47	2.3	3.41	0.374	0.05	1.48	103.6	3.0
5	4.5–6.5	14.5	1.0	0.40	2.1	3.10	0.343	0.063		20.8	
6	6.5–8.5	15.2	1.0	0.40	1.8	2.37	0.178	0.025		5.1	
7	8.5–21.0	15.0	1.0	0.40	1.9	2.76	0.217	0.026		6.2	
Emb.	–	22.0	1.0	0.50	–	–	–	–	–	–	–

[a] $k_{y(eq)} = 12\ k_{y(soil)}$.

3. Settlements analysis

The observed and predicted time-settlement behaviour on plate T is shown in Fig. 2. In accordance with Asaoka et al. [5] and Nazem et al. [9], the results show that the consideration of geometrical non-linearity tends to reduce the settlements in relation to the infinitesimal analyses, an effect which grows with time. Furthermore, the effect of updating the nodal coordinates in the infinitesimal analyses results in a slight increase of the settlements, although the effect is negligible.

The behaviour obtained with the analysis for large displacements is consistent with soil mechanics theory, since this analysis can properly simulate the dependence of the settlements with respect to the real, updated, thickness of the compressible layers by incorporating the deformation of the preceding phases. The reduced thickness of the soil is thus naturally reflected in smaller settlements.

Another factor which could induce the reduction of the settlements when the large deformation is considered is the reduction of the stress applied due to the partial submergence of the embankment. However, in this case this effect has little influence because the water table is located 0.5 m below the surface, since only one fraction remains submerged.

Fig. 3 illustrates the measured and predicted settlements at the ground surface. Similar qualitative behaviour is seen in the three numeric analyses, and for them and the observed *in situ* behaviour. The consideration of geometrical non-linearity is also found to have more influence in the zone below the foot of the embankment, that is, in the area where the largest settlements occurs, since that is where the theory of infinitesimal deformations is least realistic.

The comparison of the settlements obtained numerically and those recorded by the observation equipment placed under the embankment (Figs. 2 and 3) shows that considering geometric non-linearity in the calculation contributes to a better simulation of the "real" behaviour of the soil foundation. The differences between the numerical analyses and the measured settlements at 100 and 250 days are probably due to the consideration of equivalent coefficients of permeability, since these do not adequately simulate the "real" flow conditions of the soil foundation.

4. Horizontal displacement analysis

Fig. 4 presents the observed and predicted horizontal displacements under the foot of the main embankment at 290 and 500 days. According to the results of Asaoka et al. [5], the inclusion of geometrical non-linearity in the calculation induces small horizontal displacements relative to the infinitesimal analyses, and this difference naturally increases with time. The figure also shows that the behaviour of the embankment is qualitatively simulated by the three numerical analyses, albeit with some discrepancies. Thus, at 290 days a better simulation of the behaviour may be observed with the non-linear analysis, while at 500 days a

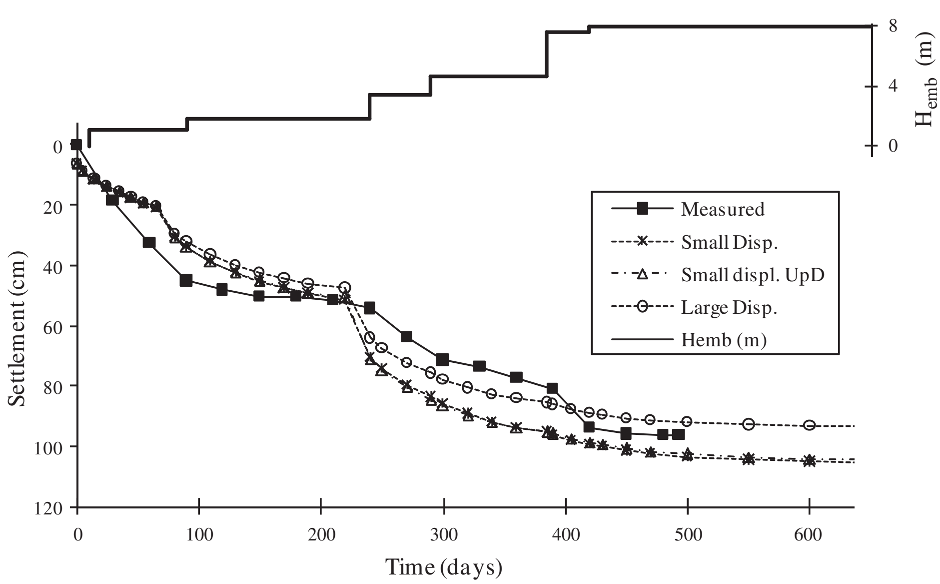
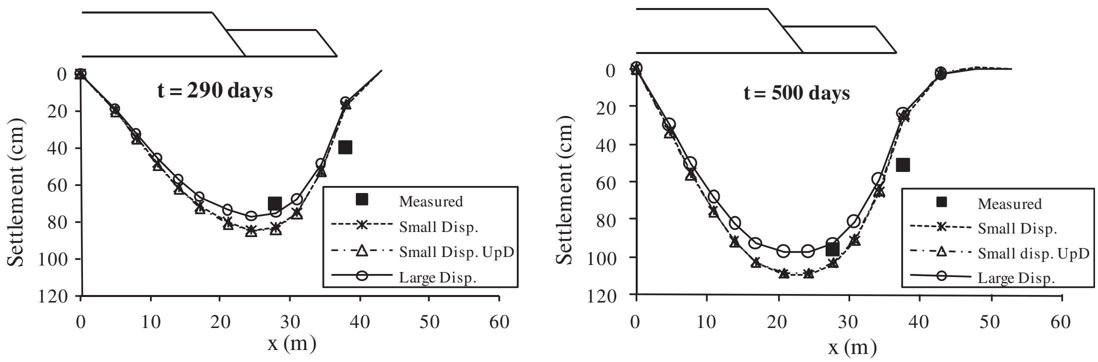


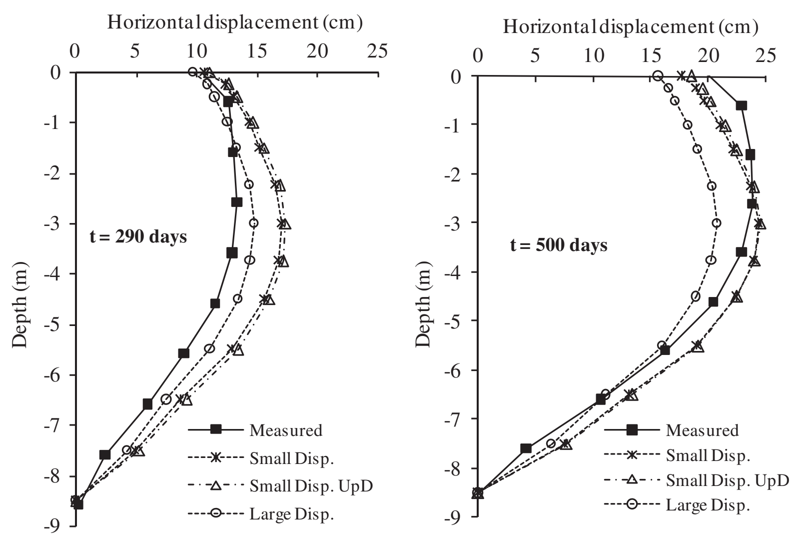
Fig. 2. Observed and predicted settlements (plate T).



(a) Evolution at 290 days.

(b) Evolution at 500 days.

Fig. 3. Observed and predicted settlements at the ground surface.



(a) Evolution at 290 days.

(b) Evolution at 500 days.

Fig. 4. Observed and predicted horizontal displacement under the foot of the embankment.

better agreement is obtained with the infinitesimal analyses. In terms of horizontal displacements, the differences between the two infinitesimal analyses (with and without updating nodal coordinates) are negligible.

Comparing the ratio between the maximum horizontal displacement (under the foot of the additional berm) and the maximum surface settlement for 2000 days, we find similar values, of 0.329, 0.325 and 0.315, for the two infinitesimal analyses (without and with updating nodal coordinates) and for large displacement analysis, respectively. Although all the numerical results are very similar, it can be concluded that large displacement analysis induces smaller displacements with a greater relative reduction in terms of horizontal displacement.

5. Pore water pressure analysis

The time evolution of the difference between the final and the initial pore pressure (u–u_0) in piezometer P is illustrated in Fig. 5. It is found that the consideration of infinitesimal analysis with updating nodal coordinates induces higher excess pore pressures immediately after a new increment of load is applied. The results show that the consideration of the updating of the nodal coordinates ("Small Disp. UpD" and "Large Disp.") leads to greater u–u_0 pressure after the consolidation phase (time >500 days).

The u–u_0 pressure at depth obtained by the two numerical analyses at 240, 420 and 2000 days is shown in Fig. 6. The results show that the analyses with updating nodal coordinates ("Small Disp. UpD" and "Large Disp.") induce higher excess pore pressures than the infinitesimal analysis without updating, with this difference increasing in the surface layers. Even at 2000 days, i.e. after the consolidation phase, u–u_0 values of 8.6 and 9.8 kPa are recorded for the analysis done with updating nodal coordinates. This is apparently inconsistent with the shortening of the drainage path which results from the decreased soil thickness, but is fully justified by the fact that these analyses consider the real nodal coordinates corrected for the deformations obtained. As the water table level does not change and the various nodal points move in the vertical direction, so the distance between the nodal points and the water table increases, therefore giving greater equilibrium pore pressures. The values of 8.6 and 9.5 kPa at 0.5 m depth correspond to the settlement of this nodal point, i.e. they are the values for the new equilibrium pore pressure.

The contours of the u–u_0 pressures, shown in Fig. 7, illustrate this clearly. An increase of the u–u_0 pressure near the surface at

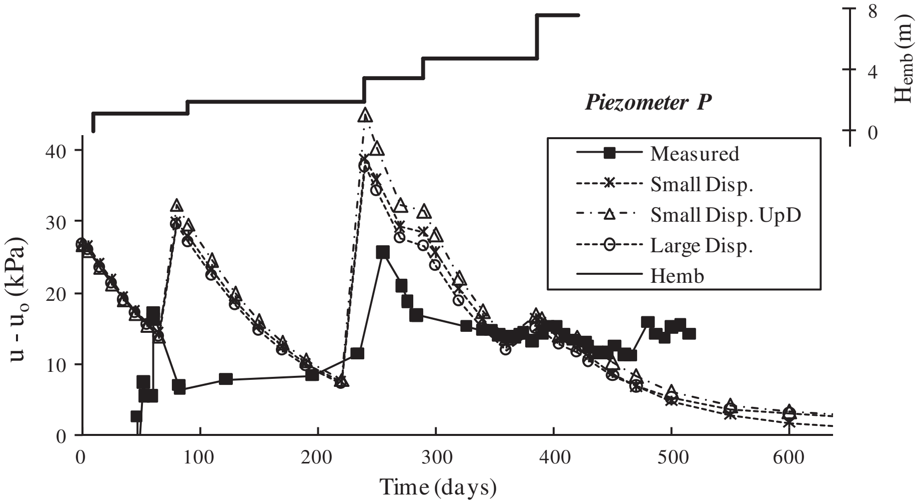


Fig. 5. Observed and predicted u–u_0 in piezometer P.

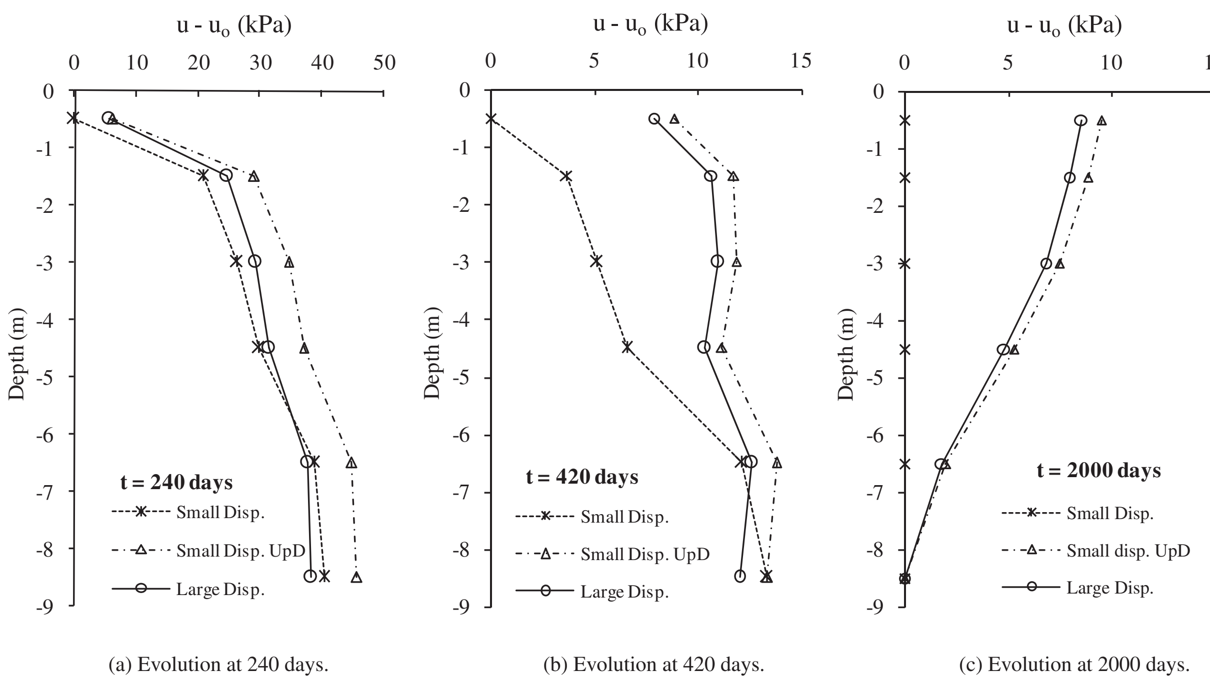


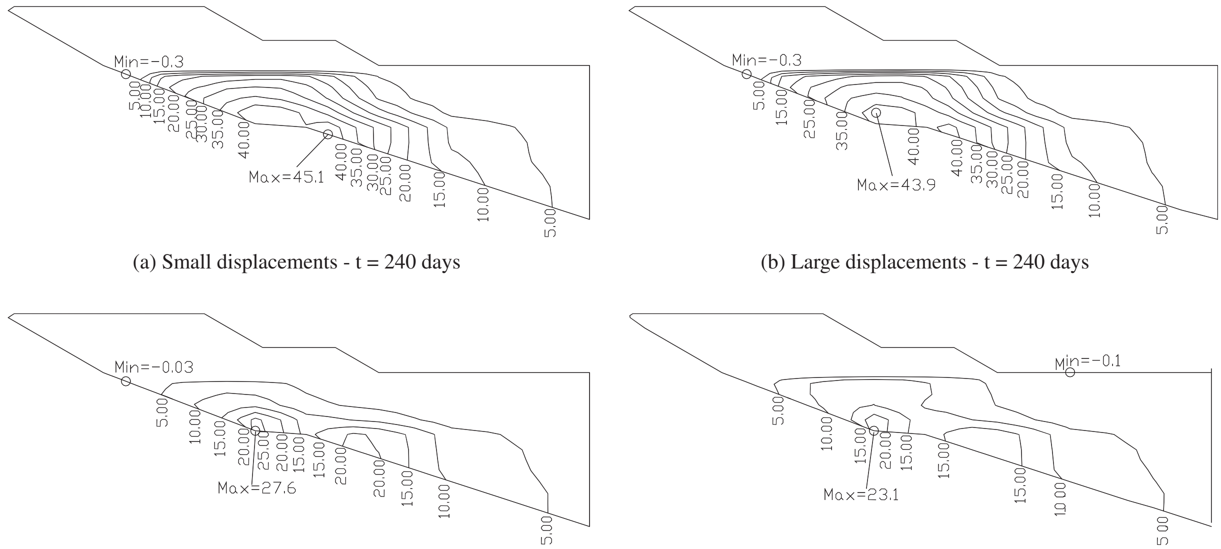
Fig. 6. Numerical predictions of u–u_0 with depth.

420 days can be seen with the geometric non-linearity, and this reflects the greatest settlement of the surface points and the corresponding increase in the equilibrium pore pressure. The reduction of the drainage path length associated with the large displacement analysis can also be seen in this figure, since this type of analysis generates lower u–u_0 pressures near the bottom boundary.

6. Conclusions

The analyses performed for this work revealed the following aspects:

– the effect of updating the nodal coordinates in the infinitesimal analysis is negligible;
– considering large displacement in the analysis leads to a reduction of settlements in relation to the infinitesimal analysis. This can be explained by the proportion of settlements in relation to the thickness of the deformable soil, so that as the soil settles the thickness of the soil layer decreases, which induces the reduction of the settlements in the subsequent phases of the calculation;
– the consideration of geometric non-linearity tends to reduce the horizontal displacements on the side of the embankment, which is also naturally associated with decreasing settlements, although the ratio of the maximum horizontal displacements to the settlements tends to increase a little;
– the time evolution of pore pressures is not particularly affected when large deformations are considered, though the pore pressures dissipate faster as time passes when large displacements are considered. This is because the progressive lessening



(a) Small displacements - t = 240 days

(b) Large displacements - t = 240 days

(c) Small displacements - t = 420 days

(d) Large displacements - t = 420 days

Fig. 7. Numerical predictions of u–u_0 contours at 240 and 420 days, with small and large displacements.

of the soil layer thickness leads to a shorter drainage path, which means that the excess pore pressures dissipate faster, leading in turn to a higher degree of consolidation. The numerical results do not show this effect clearly enough, however, since it can be masked by the increase in equilibrium pore pressure due to the greater distance between the nodal points and the water table (which level is unchanged).

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