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Fuzzy goal programming for health-care organization

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ABSTRACT

This paper presents fuzzy goal programming using with exponential membership function, which uses the modeling, and solving of health care system for optimal efficient management. The limited human resources and budget in a health-care organization are described with fuzzy conditions for determine the future strategies for unknown situations. In this study, the exponential membership function is preferred dynamic situation in next period. The study aims to assign the resources for optimization with enable management to meet the fuzzy objective of minimizing the system costs while patients are satisfied. The fuzzy goals are identified and prioritized for the strategic planning and resource allocation. A fuzzy goal-programming model is illustrated using the data provided by a health-care organization in Turkey-Sakarya private hospital.

1. Introduction

The fuzzy goal-programming model is developed and used the health-care organization for strategic planning and allocation in limited human resources. Turkey's health care system consists of the public and private sector, which are facing to very competitive conditions reason of the patients' selection independence in these days. It is facing extreme pressures to do extremely well in an environment of rapidly changing expectations, exploding global resource needs, and increased financial demands, and patients' pressure that forces to managers to give to right decisions. Furthermore, today's health-care systems are complicated by multiple objectives, multiple evaluation criteria, and evaluated by multiple decision-makers within the system, while resources and budget are extremely limited (see Tables 1 and 2).

As the health-care systems react to severe financial pressures, too much emphasis will be often placed on balancing the budget at the expense of the goals of the systems. The critical issue in the management of a health-care system is not just financial efficiency. The operational policy must be based on the compromised agreements of the diverse groups within the health-care system. Therefore, a systematic analysis and evaluation for effective resource allocation in a system are essential to provide competitive advantages for future survival and actions for the goal achievement. In this paper, a fuzzy goal-programming model is developed based on the data obtained from a private health-care organization in the Sakarya region of the Turkey. The model is analyzed and interpreted. This fuzzy goal-programming model can facilitate planning, decision-making, and managerial control by providing health-care management information. Fuzzy goal programming with exponential membership formulation for optimal resource allocation of private healthcare organization is presented.

The paper organized as follows. Section 2 presents a description of the fuzzy goal programming with exponential membership function. The main features of the proposed model construction are explained in Section 3. In next section represents the real life application and Section 5 covers the conclusion.

2. Fuzzy goal programming

Goal programming is important method for multi-objective decision making approaches in practical decision making in real life. In a standard GP formulation, goals and constraints are defined precisely but sometimes the system aim and conditions include some vague and undetermined situations. In particular, expressing the decision maker's unclear target levels for the goals mathematically and the need to optimize all goals at the same needs to complicated calculations. The fuzzy approach for goal programming tries to solve this kind of unclear difficulties.

This study includes one than more goals to optimize the resource allocation. Goal programming preferred due to realize two or more aim in the system. It is a kind of the multi criteria decision making problem which includes the crisp and vague values.

First time fuzzy set defined mathematically by Zadeh (1965) with the assigning to each possible element in the universe of

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0360-8352/

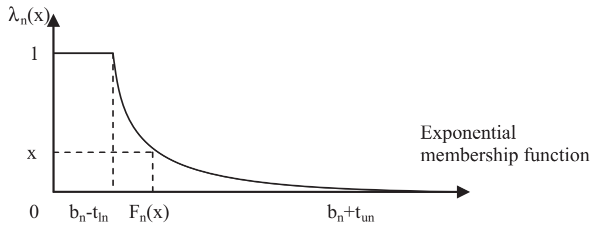

Fig. 1. Exponential membership function type for the minimization objectives.

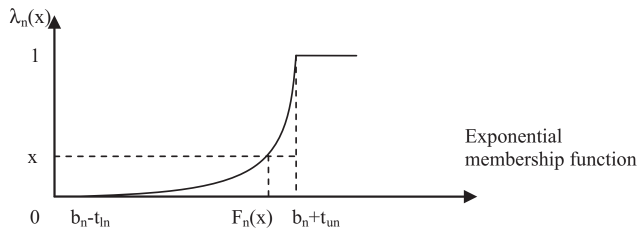

Fig. 2. Exponential membership function type for the maximization objectives.

discourse a value representing its grade of membership in the fuzzy set. This grade corresponds to the degree to which that element is similar to the concept represented by the fuzzy set. So elements may belong in the fuzzy set to a greater or lesser degree, which indicated by a larger or smaller membership grade. These membership grades are very often represented by real number values ranging in the closed interval between 0 and 1. Bellman and Zadeh (1970) mentioned the decision making in fuzzy environment. Zimmermann (1978) reviewed the fuzzy programming and linear programming with several objective functions and introduced the field of multi-objective optimization problems. Ahn (2015) presented the simple method for finding the extreme points of various types of incomplete attribute weights. Also Slowinski (1986) applied fuzzy linear programming method to water supply system development planning.

A goal that is not completely achieved has an under-achievement (negative deviation) or overachievement (positive deviation) of the goal. If the objective is to exceed stated goals, the objective function will only contain a negative deviational variable, $d-$. If the objective is to be under the stated goal, the objective function will contain a positive deviational variable, *d+*.

In real life applications are used to somewhere by researchers, such as Chen and Tsai (2001) used to capacity allocation and choice problem, Jamalnia and Soukhakian (2009) developed aggregate production planning for a medium range capacity planning, Biswas and Pal (2005) presented low fuzzy goal programming can be effectively used for modeling and solving land use planning problems in agricultural systems for optimal production of several seasonal crops in a planning year. Tsai, You, Lin, and Tsai (2008) presented to address a steel supplier's channel allocation problem that includes decisions of channel mix and capacity allocation for each distribution channel with fuzzy goal programming approach. Also Kumar, Vrat, and Shankar (2004) used the fuzzy goal programming to vendor selection problem in supply chain. Zeng, Kang, Li, Zhang, and Guo (2010) applied to fuzzy multi objective linear programming to crop and planning in a fuzzy environment. Kwak and Lee (1997) suggested the linear goal programming for human resource allocation in a health care organization. Also Romero (1986) generalized the goal programming approach. Khalili-Damghani, Sadi-Nezhad, and Tavana (2013) applied to fuzzy goal programming to the project selection problems with TOPSIS and a fuzzy preference relation. In our study we preferred to the Pareto analysis.

Li and Hu (2009) proposed satisfying optimization method based on goal programming for fuzzy multiple objective optimization problem. Chen and Tsai (2001) suggested the fuzzy goal programming with different importance and priorities to capacity allocation and choice problem. Also Liang proposed the fuzzy multi-objective Project management decisions using two-phase fuzzy goal programming approach. Also, Baky developed a new algorithm for solving decentralized bi-level multi-objective programming (DBL-MOP) problems with a single decision maker at the upper level and multiple decision makers at the lower level. Wang and Li derived interval weights on fuzzy preference relations to goal programming. Jimenez and Bilbao represented the pareto-optimal solutions in fuzzy multi-objective linear programming. Mehrjerdi (2011) suggested to solving fractional programming through fuzzy goal setting and approximation. Sakawa and Matsui (2012) used random variables in two-level linear programming with Stackelberg solutions. Gong, Li, Zhou, and Yao (2009) suggested the priority vectors from the intuitionistic fuzzy preference relations in goal programming approach. Silva and Marins (2014) suggested for solving aggregate production-planning problems under uncertainty.

However in contrast to LP and GP approaches, fuzzy programming (FP) approach to resource allocation and efficiency usage in health care organization problems has not been appeared extensively in the literature. In this paper, fuzzy goal programming formulation for optimal resource allocation and usage is presented for health care organization.

$$\left.\begin{array}{ll} \text{Find } x_i, & i = 1,\ldots,n \\ Z_m(x_i) \prec \overline{Z}_m & m = 1,\ldots,M \\ Z_k(x_i) \prec \overline{Z}_k & k = M+1,\ldots,K \\ g_j(x_i) \leqslant b_j & j = 1,\ldots,J \\ x_i \geqslant 0 & i = 1,\ldots,n \end{array}\right\} \quad (1)$$

where $Z_m(x_i)$ is the mth goal constraint, $Z_k(x_i)$ is the kth goal constraint, $\bar{Z}_m(x_i)$ is the target value of mth goal, $\bar{Z}_k(x_i)$ is the target value of the kth goal, $g_j(x_i)$ is the jth inequality constraint and b_j is the available resource of inequality constraint j (Zimmermann, 1978).

In formulation (1) the symbols "$\prec$ and $\succ$" denote the fuzzified versions of "$\leqslant$ and $\geqslant$ and can be read as "approximately less/greater than or equal to". These two types of linguistic terms have different meanings. Under "approximately less than or equal to" situation, the goal m is allowed to be spread to the right-hand side of $\bar{Z}_m(\bar{Z}_m = l_m$ where l_m denote the lower bound for the mth objective) with a certain range of $r_m(\bar{Z}_m + r_m = u_m$, where l_m denote the upper bound for the mth objective). Similarly, with "approximately greater than or equal to", p_k is the allowed left side of $\bar{Z}_k(\bar{Z}_k + r_m = l_k$ and $\bar{Z}_k = u_k)$.

In this paper, an exponential, instead of linear membership function is proposed. The fuzzy goals are characterized by exponential membership function with defining the lower or upper tolerance limit (see Figs. 1 and 2). The advantages of using exponential membership are twofold. First, the resulting problems can be transformed to linear ones when the "product" and several other nonlinear aggregate operators are used. Secondly, exponential representation is more realistic than the linear ones usually used for some practical applications. It depends on the fuzzy restriction given to a fuzzy goal of the problem in a fuzzy decision-making situation. Let t_{ln} and t_{un} be the lower- and upper-tolerance ranges considered respectively, for achievement of the aspired level b_n of the nth fuzzy goal. Then, the exponential membership function $\mu_n(X)$, for the fuzzy goal $F_n(X)$ can be characterized the lower tolerance limit $(b_n - t_{ln})$ and upper tolerance limit $(b_n + t_{un})$ are presented as follows (Pal, Moitra, & Maulik, 2003):

$$\mu_n(X) = \begin{cases} 1 & if \quad F_n(X) \geqslant b_n \\ \frac{e^{-\alpha_i(b_n - F_n(x))/t_{un}} - e^{-\alpha_i}}{1-e^{-\alpha_i}} & if \quad b_n - t_{ln} \leqslant F_n \leqslant b_n \\ 0 & if \quad F_n(X) < b_n - t_{ln} \end{cases} \quad (2)$$

and

$$\mu_n(X) = \begin{cases} 1 & if \quad F_n(X) \leqslant b_n \\ 1 - \frac{e^{-\alpha_i} - e^{-\alpha_i(b_n - F_n(x))/t_{ln}}}{1-e^{-\alpha_i}} & if \quad b_n \leqslant F_n \leqslant b_n + t_{un} \\ 0 & if \quad F_n(X) > b_n + t_{un} \end{cases} \quad (3)$$

The exponential membership function based fuzzy goal programming with upper and lower level conditions can be presented as follows:

Maximize λ

Subject to

$$\frac{e^{-\alpha_i(b_n - F_n(x))/t_{un}} - e^{-\alpha_i}}{1 - e^{-\alpha_i}} \leqslant \lambda \quad n = 1,2,\ldots,N \quad (4)$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,2,\ldots,N; \quad \sum_{i=1}^{n} x_{ij} = 1, \quad i = 1,2,\ldots,N; \quad \lambda \geqslant 0, \quad (5)$$

$$x_{ij} = \begin{cases} 1, & \text{if the ith resource is assigned to the jth task} \\ 0, & \text{if the ith resource is not assigned to the jth task} \end{cases} \quad (6)$$

Minimize λ

Subject to

$$\frac{e^{-\alpha_i} - e^{-\alpha_i(b_n - F_n(x))/t_{ln}}}{1 - e^{-\alpha_i}} > \lambda \quad n = 1,2,\ldots,N \quad (7)$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,2,\ldots,N; \quad \sum_{i=1}^{n} x_{ij} = 1, \quad i = 1,2,\ldots,N; \quad \lambda \geqslant 0, \quad (8)$$

$$x_{ij} = \begin{cases} 1, & \text{if the ith resource is assigned to the jth task} \\ 0, & \text{if the ith resource is not assigned to the jth task} \end{cases} \quad (9)$$

Then, in the goal achievement function, the under-deviational variables are minimized on the basis of importance of achieving the aspired goal levels in the decision-making context.

The fuzzy goal-programming model of the problem under a preemptive priority structure can be presented as follows:

$$\begin{aligned} &Minimize \quad Z = [P_1(d^-), P_2(d^-), \ldots, P_i(d^-)] \\ &\frac{e^{-\alpha_i(b_n - F_n(x))/t_{un}} - e^{-\alpha_i}}{1 - e^{-\alpha_i}} + d_n^- - d_n^+ = 1 \\ &1 - \frac{e^{-\alpha_i(b_n - F_n(x))/t_{un}} - e^{-\alpha_i}}{1 - e^{-\alpha_i}} + d_n^- - d_n^+ = 1 \\ &d_n^-, d_n^+ \geqslant 0 \quad n = 1,2,\ldots,N \end{aligned} \quad (10)$$

where Z represents the vector of i priority achievement functions and d_n^-, d_n^+ are the under- and over-deviational variables of the nth goal. $P_i(d_i^-)$ is a linear function of the weighted under-deviational variables, where $P_i(d_i^-)$ is of the form

$$P_i(d^-) = \sum_{n=1}^{N} \omega_{in}^- d_{in}^-, d_{in}^- \geqslant 0, \quad (n = 1,2,\ldots,N) \quad (11)$$

where d_{in}^- is renamed for d_n^- to represent it at the ith priority level, ω_{in}^- is the numerical weight associated with d_{in}^- and represents the weight of importance of achieving the aspired level of the nth goal relative to others which are grouped together at the ith priority level. The ω_{in}^- values are determined as (Zimmermann, 1987): d_{in}^-

$$\omega_{ik}^- = \begin{cases} \frac{1}{(t_{ln})_i} & \text{for the defined } \mu_n(X) \text{ in } (1) \\ \frac{1}{(t_{un})_i} & \text{for the defined } \mu_n(X) \text{ in } (1) \end{cases} \quad (12)$$

where $(t_{ln})_i$ and $(t_{un})_i$ are used to represent t_{lk} and t_{uk}, respectively, at the ith priority level.

It is worthy to mention here that the notion of preemptive priorities of the goals actually holds that the $_i$th priority P_i is preferred to the next priority P_{i+1} regardless of any multiplier associated with P_{i+1}.

In the decision-making situation, exponential membership goals with highest membership value (unity) as their achievement levels are defined for the exponential membership functions of the fuzzy goals of the problem on the origin of the priorities of importance of achieving the desired levels of the fuzzy goals to the range possible is considered. In M-Pareto optimal solution denote the different trade-off requirements among the objectives (Sakawa, Kato, & Katagiri, 2004).

3. Model construction

Consider a health care system that organization serves N types of tasks to satisfy customer demands over in i service in planning horizon T. The purposes are to determine overall service levels for each task category to meet the changeable or uncertain number of patients in near future and to make decisions and adopt policies on the issues of hiring, lay off, overtime, subcontract and inventory. As a dynamic structure the previous term and current term system variable values are considered. The system parameters defined and problem statement and assumptions given in below.

3.1. Parameters definition

Index sets

i	index for service type, for all $i = 1,2,\ldots,I$
n	index for task type, for all $n = 1, 2,\ldots,N$
h	index for planning time period, for all $h = 1,2,\ldots,H$
g	index for objective, for all $g = 1,2,\ldots,K$
t	index for time

Decision variables

p_i	number of in-patient stays in each service
o_i	number of an operations in each service
D_i	demand of each service
U_i	capacity of each service
P_i	total budget
F	flexibility of the service quota allocation
B_i	number of patient target of each service
$W1_{ti}$	number of physician in each service in t period
$W2_{ti}$	number of nurses in each service in t period
$W3_{ti}$	number of technician in each service in t period
a_{it}	arrived patient in each service in t period
CI_i	investment and maintenance cost in each service
$CS1_i$	physician's salary in department i in period t
$CS2_i$	nurse's salary in department i in period t
$CS3_i$	technician's salary in department i in period t
CB_i	cost of building in each service
CE_i	cost of equipment and technology investment in each service
CM_i	medication cost (per patient) in each service
SC_i	supplier cost in each service
$A1_{1i}$	the number of service tasks that one physician can produce in one month on regular time in i service
$A2_{1i}$	the number of service tasks that one nurse can produce in one month on regular time in i service
$A3_{1i}$	the number of service tasks that one technician can produce in one month on regular time in i service
$A1_{2i}$	the maximum number of service tasks that one physician can produce in one month on over time in i service
$A2_{2i}$	the maximum number of service tasks that one nurse can produce in one month on over time in i service
$A3_{2i}$	the maximum number of service tasks that one

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Table 1
Decision variables values for objective functions.

Variable	Department	Sub departments	Demand per month (D_{it})	Capacity per month (U_{it})	Total budget per month (P_{it})	Flexibility of the service (F_{it})	Number of patient target of each service per month (B_{it})
X_1	General Surgery	$X_{1,1}$ Laparoscopic Surgery	82	95			85
		$X_{1,2}$ Gastrointestinal Surgery	23	25			23
		$X_{1,3}$ Endocrinology Surgery	42	40			36
		$X_{1,4}$ Vascular Surgery	12	15			14
		Sum	159	175	988,476	15%	158
X_2	Orthopedics and Traumatology	$X_{2,1}$ Trauma Surgery	34	35			25
		$X_{2,2}$ Orthopedics Oncology	56	60			50
		$X_{2,3}$ Joint Prosthesis Surgery	67	70			63
		$X_{2,4}$ Vertebral Surgery	54	55			50
		$X_{2,5}$ Shoulder Surgery	73	75			68
		$X_{2,6}$ Sport Injuries and Arthroscopic Surgery	34	35			32
		$X_{2,7}$ Pediatric Orthopedics	67	70			63
		Sum	385	400	904,686	22%	351
X_3	Obstetrics and Gynecology	$X_{3,1}$ Pregnancy Follow-up and Delivery	89	90			80
		$X_{3,2}$ General Gynecology	76	75			68
		$X_{3,3}$ Gynecologic Oncology	45	50			45
		$X_{3,4}$ Infertility IVF	86	85			77
		$X_{3,5}$ Menopause	52	55			50
		$X_{3,6}$ Urogynaecology	56	60			45
		Sum	404	415	1,372,958	10%	365
X_4	Urology	$X_{4,1}$ Urology Oncology	67	60			55
		$X_{4,2}$ Prostate Diseases (Tur, Holmium Laser)	43	45			42
		$X_{4,3}$ Urinary System Stone Therapy (ESWL, Laser, Surgical)	76	75			68
		$X_{4,4}$ Male Genital Organ Diseases and Infertility Treatment	34	35			32
		Sum	220	215	1,010,975	15%	197
X_5	Internal Medicine	$X_{5,1}$ General Medicine	23	20			18
		$X_{5,2}$ Gastroenterology	45	50			45
		$X_{5,3}$ Oncology	67	65			59
		$X_{5,4}$ Rheumatology	34	30			27
		$X_{5,5}$ Endocrinology	56	50			45
		$X_{5,6}$ Nephrology	34	30			27
		$X_{5,7}$ Dialysis Center	24	25			23
		Sum	283	270	1,398,182	20%	244
X_6	Cardiology		98	90			700
		Sum	98	90	1,186,834	10%	82
X_7	Pediatrics	$X_{7,1}$ General Pediatrics	56	60			45
		$X_{7,2}$ Pediatric Oncology	34	40			28
		$X_{7,3}$ Pediatric Gastroenterology	23	20			20
		$X_{7,4}$ Pediatric Cardiology	56	50			45
		$X_{7,5}$ Pediatric Hematology	78	80			75
		$X_{7,6}$ Pediatric Neurology	45	50			45
		$X_{7,7}$ Pediatric Endocrinology	78	80			70
		$X_{7,8}$ Pediatric Asthma and Allergic Diseases	84	85			77
		Sum	454	465	1,664,511	15%	405
X_8	Neurology		56	50			650
		Sum	56	50	1,232,488	15%	650
X_9	Psychiatry and Psychology	$X_{9,1}$ Adult Psychiatry and Psychology	45	50			50
		$X_{9,2}$ Adolescent and Pediatric Psychiatry	34	35			90
		$X_{9,3}$ Group Therapy (panic attacks, phobias etc.)	57	60			110
		Sum	136	145	895,637	25%	250
X_{10}	Cardiovascular Surgery	X_{10} Cardiovascular Surgery	34	40			500
		Sum	34	40	1,061,813	5%	500
X_{11}	Pediatric Surgery	$X_{11,1}$ Pediatric Urology	76	75			650
		Sum	76	75	1,414,934	20%	650
X_{12}	Ear-Nose-Throat	$X_{12,1}$ Head and Neck Surgery	67	70			65
		$X_{12,2}$ Neurotology	67	65			65
		$X_{12,3}$ Rhinology (Endoscopic Surgery)	45	50			50
		$X_{12,4}$ Phoniatry and Voice Diseases	34	35			35

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Table 1 (*continued*)

Variable	Department	Sub departments	Demand per month (D_{it})	Capacity per month (U_{it})	Total budget per month (P_{it})	Flexibility of the service (F_{it})	Number of patient target of each service per month (B_{it})
		$X_{12,5}$ Pediatric E.N.T.	78	75			70
		$X_{12,6}$ Microsurgery	54	60			45
		Sum	345	355	684,710	20%	330
X_{13}	Ophthalmology	$X_{13,1}$ Uveitis and Infectious Diseases	32	40			90
		$X_{13,2}$ Cataract Surgery	65	60			115
		$X_{13,3}$ Retina Surgery	87	85			120
		$X_{13,4}$ Glaucoma Surgery	98	95			140
		$X_{13,5}$ Strabismus Surgery	56	60			130
		$X_{13,6}$ Laser Therapis	34	35			80
		Sum	372	375	1,210,802	25%	675
X_{14}	Intensive Care Units	$X_{14,1}$ Surgical Intensive Care Unit	78	75			70
		$X_{14,2}$ Neonatal Intensive Care Unit	45	50			40
		$X_{14,3}$ Coronary Intensive Care Unit	87	90			45
		$X_{14,4}$ Open Heart Surgery Intensive Care Unit	45	50			45
		$X_{14,5}$ Intensive Care Unit	78	75			50
		Sum	333	340	1,173,698	5%	250
X_{15}	Dermatology	$X_{15,1}$ Esthetic Dermatology	67	65			60
		$X_{15,2}$ Dermatologic Laser Clinic	45	50			40
		$X_{15,3}$ UV Therapy	87	90			70
		$X_{15,4}$ Dermatoscopy	70	75			70
		$X_{15,5}$ Chemical Peeling	56	60			55
		Sum	325	340	895,035	10%	295
X_{16}	Physical Therapy and Rehabilitation	$X_{16,1}$Vertebral Diseases	45	50			45
		$X_{16,2}$ Joint Diseases	67	70			60
		$X_{16,3}$ Osteoporosis and Rheumatologic Diseases	34	45			45
		$X_{16,4}$ Muscle Diseases	67	70			60
		$X_{16,5}$ Postoperative Rehabilitation	45	40			35
		$X_{16,6}$ Rehabilitation of Stroke Patients	34	35			30
		$X_{16,7}$ Physical Therapy and Rehabilitation	76	80			70
		Sum	368	390	1,431,004	20%	345
X_{17}	Emergency	$X_{17,1}$ Emergency	982	1000			10000
		Sum	982	1000	1,111,360	10%	10000
X_{18}	Dental Clinic	$X_{18,1}$ Dental Clinic	342	350			325
		Sum	342	350	718,270	15%	325

technician can produce in one month on over time in i service

$A1_{4i}$ the desired physician number at the end of the planning horizon in i service

$A2_{4i}$ the desired nurse number at the end of the planning horizon in i service

$A3_{4i}$ the desired technician number at the end of the planning horizon in i service

A_{5i} the desired investment level at the end of each month in i service

Y_i number of patient complaints

m the number of months in the planning horizon

3.2. Problem statement and assumptions

Objective functions are both quantitative and qualitative. In this study objective and goal have the same meaning and are used in a substitution manner. Qualitative objectives are stated with linguistic terms. Quantitative objective functions are:

1. Minimize total service costs,

$$Z_1 = \sum_{i=1}\sum_{t=1}(W1_{ti}) * (CS1_i) + (W2_{ti}) * (CS2_i) + (W3_{ti}) * (CS3_i) + (p_i * (CM_i) + (SC_i)) + CI_i + CB_i + CE_i$$

2. Minimize total investment costs,

$$Z_2 = \sum_{i=1}\sum_{t=1}\frac{CI_{ti}}{a_i}$$

3. Minimize the current resource usage

$$Z_3 = \sum_{i=1}\sum_{t=1}\frac{W1_{ti}}{a_i}, \frac{W2_{ti}}{a_i}, \frac{W3_{ti}}{a_i}$$

4. Minimize the rate of changes in workforce

$$Z_4 = \sum_{i=1}\sum_{t=1}\frac{W1_{ti}}{W1_{t-1i}}, \frac{W2_{ti}}{W2_{t-1i}}, \frac{W3_{ti}}{W3_{t-1i}}$$

5. Minimize the patient complaints level

$$Z_5 = \frac{Y_i}{a_i}$$

Table 2
Decision variables values for constraints.

Variable	$W1_{(t-1)i}$ Number of physician in $t-1$ period	$W2_{(t-1)i}$ Number of nurses in $t-1$ period	$W3_{(t-1)i}$ Number of technician in $t-1$ period	$W1_{ti}$ Number of physician in t period	$W2_{ti}$ Number of nurses in t period	$W3_{ti}$ Number of technician in t period	a_{it} Arrived patient in t period	CI_i Investment and maintenance cost
X_1	5	4	1	4	3	1	1345	548
X_2	2	3	1	3	2	0	3406	3404
X_3	3	2	1	4	3	0	3592	4529
X_4	4	4	0	2	2	1	2036	4738
X_5	2	4	1	5	3	1	4207	1519
X_6	1	3	1	3	1	0	3476	6934
X_7	4	4	0	2	1	0	4977	2120
X_8	2	4	1	2	2	0	204	4602
X_9	4	3	1	3	2	1	2381	350
X_{10}	2	4	2	2	2	0	3900	661
X_{11}	5	5	1	4	2	1	4800	343
X_{12}	3	4	2	2	2	0	2875	6013
X_{13}	2	3	1	3	1	0	4537	4968
X_{14}	4	2	1	4	3	0	3592	4529
X_{15}	5	4	0	2	2	1	2036	4738
X_{16}	3	2	1	5	3	1	4207	1519
X_{17}	2	4	0	3	2	0	3476	6934
X_{18}	5	2	0	4	1	0	4977	2120

Variable	$CS1_i$ Physician's salary in department i in period t	$CS2_i$ Nurse's salary in department i in period t	$CS3_i$ Technician's salary in department i in period t	CB_i Cost of building	CE_i Cost of equipment and technology investment	CM_i Medication cost (per patient)	SC_i Supplier cost	p_i Number of in-patient stays	r_i Cost of care in-patient stays	Y_i Patient complaints
X_1	64,500	42,500	45,000	6000	22,500	40	12	125	499,476	56
X_2	67,500	42,500	45,000	6500	28,000	50	16	343	413,180	45
X_3	75,000	42,500	45,000	5500	27,000	60	67	321	578,555	3
X_4	65,000	42,500	45,000	4500	11,500	50	97	280	380,535	25
X_5	70,000	42,500	45,000	4000	16,000	70	56	236	607,428	56
X_6	75,000	42,500	45,000	6000	20,000	60	88	681	489,623	28
X_7	64,500	42,500	45,000	5000	22,500	55	90	529	324,409	12
X_8	65,000	42,500	45,000	4450	17,000	65	45	648	375,198	78
X_9	65,000	42,500	45,000	4625	12,000	60	30	232	382,627	45
X_{10}	75,000	42,500	45,000	2250	17,000	50	50	546	334,713	23
X_{11}	67,500	42,500	45,000	4625	12,000	60	30	642	494,464	45
X_{12}	65,000	42,500	45,000	3900	11,000	50	44	175	321,378	12
X_{13}	67,500	42,500	45,000	4500	17,000	70	23	675	400,455	19
X_{14}	75,000	42,500	45,000	3250	10,500	60	45	234	524,858	72
X_{15}	65,000	42,500	45,000	3500	2250	55	75	243	350,465	26
X_{16}	70,000	42,500	45,000	4000	6000	65	12	455	592,727	6
X_{17}	75,000	42,500	45,000	3350	10,000	60	45	10000	1,456,035	42
X_{18}	64,500	42,500	45,000	2500	7000	50	12	434	367612	45

Based on the characteristics of healthcare organization management decision problem is constructed as the mathematical model which based on the following assumptions:

1. The exponential membership functions are constructed to each of the quantitative and qualitative objectives based on decision maker's judgments.
2. Objectives have different priorities and used to maximize the summation of achievement degrees of all fuzzy objectives.
3. Actual workforce level, service capacity and bed space in each period cannot exceed from their maximum levels.

3.3. Constraints

Constraint 1: a constraint on the demand of services

$$\sum_{i=1}^{n} x_i = D \quad (when \quad D \prec U) \quad or \quad \sum_{i=1}^{n} x_i = U (when \quad D \succ U)$$

Constraint 2: a constraints on the maximum capacity of each service

$$x_i \leqslant U_i \quad for \quad i = 1, 2, 3$$

Constraint 3: a constraint on the total budget

$$\sum_{i=1}^{n} r_i x_i \leqslant P$$

Constraint 4: a constraint on the flexibility of the service quota allocation

$$\sum_{i=1}^{n} f_i x_i \geqslant F$$

Constraint 5: constraints on the number of patient target of each service

$$p_i x_i \leqslant B_i \quad for \quad i = 1, 2, 3$$

Constraint 6: all allocation quantities are nonnegative

$$x_i \geqslant 0$$

In this real case, linear membership functions are given in Eqs. (3)–(6). The aspiration levels of the five fuzzy goals are obtained total service cost = 9,000,000 (per six month), total investment cost = 300,000 (per six month), the current resource usage = 0.002, rate of changes in workforce = 1 for physicians, 0.7 for nurses and 0.5 for technicians, each patient complaints rate = 0.002.

GOAL I-Minimize total service costs

$$\mu_1(X) = \begin{cases} 1 & if \ Z_1(X) < 8{,}586{,}172 \\ \frac{e^{-\alpha_i (9{,}000{,}000 - Z_1(X))/9{,}000{,}000 - 8{,}586{,}172} - e^{-\alpha_i}}{1 - e^{-\alpha_i}} & if \ 8{,}586{,}172 < Z_1(X) < 9{,}000{,}000 \\ 0 & if \ Z_1(X) > 9{,}000{,}000 \end{cases}$$

GOAL II-Minimize total investment costs

$$\mu_2(X) = \begin{cases} 1 & if \ Z_2(X) < 0.00174 \\ \frac{e^{-\alpha_i (0.002 - Z_2(X))/0.002 - 0.00174} - e^{-\alpha_i}}{1 - e^{-\alpha_i}} & if \ 0.00174 < Z_2(X) < 0.0020 \\ 0 & if \ Z_2(X) > 0.002 \end{cases}$$

GOAL III-Minimize the current resource usage

$$\mu_3(X) = \begin{cases} 1 & if \ Z_3(X) < 289{,}250 \\ \frac{e^{-\alpha_i (300{,}000 - Z_3(X))/300{,}000 - 289{,}250} - e^{-\alpha_i}}{1 - e^{-\alpha_i}} & if \ 289{,}250 < Z_3(X) < 300{,}000 \\ 0 & if \ Z_3(X) > 300{,}000 \end{cases}$$

GOAL IV-Minimize the rate of changes in workforce

$$\mu_{41}(X) = \begin{cases} 1 & if \quad Z_{41}(X) < 0.9841 \\ \frac{e^{-\alpha_i(1-Z_{41}(x))/1-0.9841} - e^{-\alpha_i}}{1-e^{-\alpha_i}} & if \quad 1 < Z_{41}(X) < 0.9841 \\ 0 & if \quad Z_{41}(X) > 1 \end{cases}$$

$$\mu_{42}(X) = \begin{cases} 1 & if \quad Z_{42}(X) < 0.6712 \\ \frac{e^{-\alpha_i(0.7-Z_{42}(x))/0.7-0.6212} - e^{-\alpha_i}}{1-e^{-\alpha_i}} & if \quad 0.6712 < Z_{42}(X) < 0.70 \\ 0 & if \quad Z_{42}(X) > 0.7 \end{cases}$$

$$\mu_{43}(X) = \begin{cases} 1 & if \quad Z_{43}(X) < 0.4705 \\ \frac{e^{-\alpha_i(0.5-Z_{43}(x))/0.5-0.4705} - e^{-\alpha_i}}{1-e^{-\alpha_i}} & if \quad 0.5 < Z_{43}(X) < 0.4705 \\ 0 & if \quad Z_{43}(X) > 0.5 \end{cases}$$

GOAL V-Minimize the patient complaints level

$$\mu_{5}(X) = \begin{cases} 1 & if \quad Z_{5}(X) < 0.0101 \\ \frac{e^{-\alpha_i(0.02-Z_{5}(x))/0.02-0.0101} - e^{-\alpha_i}}{1-e^{-\alpha_i}} & if \quad 0.0101 < Z_{5}(X) < 0.2 \\ 0 & if \quad Z_{5}(X) > 0.2 \end{cases}$$

4. Objective function

The health care organization resource planning model with fuzzy goal programming approach is formulated as follows:

Maximize $f(u) = \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5$

Minimize total service costs $\mu_1 : 21739 - 0.002415 \left(\sum_{i=1}\sum_{t=1}(W1_{ti}) * (CS1_i) + (W2_{ti}) * (CS2_i) + (W3_{ti}) * (CS3_i) + (p_i * (CM_i) + (SC_i)) + CI_i + CB_i + CM_i\right) + d1^- - d1^+ = 1$

Minimize total investment costs

$\mu_2 : 7.69 - 3846.15(\sum_{i=1}\sum_{t=1}\frac{CI_{ti}}{a_i}) + d_2^- - d_2^+ = 1$

Minimize the current resource usage

$\mu_3 : 27,907 - 9.3 \cdot 10^{-5}\left(\sum_{i=1}\sum_{t=1}\frac{W1_{ti}}{a_i}, \frac{W2_{ti}}{a_i}, \frac{W3_{ti}}{a_i}\right) + d_3^- - d_3^+ = 1$

Minimize the rate of changes in workforce

$\mu_{41} : 1.0162 - 1.0162\left(\sum_{i=1}\sum_{t=1}\frac{W1_{ti}}{W1_{t-1i}}\right) + d_{41}^- - d_{41}^+ = 1$

$\mu_{42} : 8.883 - 12,960\left(\sum_{i=1}\sum_{t=1}\frac{W2_{ti}}{W2_{t-1i}}\right) + d_{42}^- - d_{42}^+ = 1$

$\mu_{43} : 16.95 - 2.125\left(\sum_{i=1}\sum_{t=1}\frac{W3_{ti}}{W3_{t-1i}}\right) + d_{43}^- - d_{43}^+ = 1$

Minimize the patient complaints level

$\mu_5 : 2.02 - 101.01\left(\frac{Y_i}{a_i}\right) + d_5^- - d_5^+ = 1$

Constraint 1:

$$159X_1 + 385X_2 + 404X_3 + 220X_4 + 283X_5 + 98X_6 + 454X_7 + 56X_8 + 136X_9 + 34X_{10} + 76X_{11} + 345X_{12} + 372X_{13} + 333X_{14} + 325X_{15} + 368X_{16} + 982X_{17} + 342X_{18} \leqslant 6500$$

Constraint 2:

$$178X_1 + 400X_2 + 415X_3 + 215X_4 + 270X_5 + 90X_6 + 465X_7 + 50X_8 + 145X_9 + 40X_{10} + 75X_{11} + 355X_{12} + 375X_{13} + 340X_{14} + 340X_{15} + 390X_{16} + 1000X_{17} + 325X_{18} \leqslant 6500$$

Constraint 3:

$$499,476X_1 + 413,180X_2 + 578,555X_3 + 380,535X_4 + 607,428X_5 + 489,623X_6 + 324,409X_7 + 375,198X_8 + 382,627X_9 + 334,713X_{10} + 494,464X_{11} + 321,378X_{12} + 400,455X_{13} + 524,858X_{14} + 350,465X_{15} + 592,727X_{16} + 445,515X_{17} + 367,612X_{18} \leqslant 9,000,000$$

Constraint 4:

$$0.15X_1 + 0.22X_2 + 0.10X_3 + 0.15X_4 + 0.20X_5 + 0.10X_6 + 0.15X_7 + 0.15X_8 + 0.25X_9 + 0.05X_{10} + 0.20X_{11} + 0.20X_{12} + 0.25X_{13} + 0.05X_{14} + 0.10X_{15} + 0.20X_{16} + 0.10X_{17} + 0.15X_{18} \geqslant 0.10$$

Constraint 5:

$$125X_1 + 343X_2 + 321X_3 + 280X_4 + 236X_5 + 681X_6 + 529X_7 + 648X_8 + 232X_9 + 546X_{10} + 642X_{11} + 175X_{12} + 675X_{13} + 234X_{14} + 243X_{15} + 455X_{16} + 376X_{17} + 434X_{18} \leqslant 16,938$$

Constraint 6:

$$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16}, X_{17}, X_{18} \geqslant 0$$

Following the proposed procedure executed using the software LINGO (Ver 11.0). The optimal solution for the above formulation is $\mu_1(Z_1)$ = 9,001,242, $\mu_2(Z_2)$ = 0.1739, $\mu_3(Z_3)$ = 0.3, $\mu_{4,1}(Z_{4,1})$ = 0.1594 $\mu_{4,2}(Z_{4,2})$ = 0.6083, $\mu_{4,3}(Z_{4,3})$ = 7.506 $\mu_5(Z_5)$ = 0.1009.

Assume that the initial value of the degree α to be 0.5 and the initial reference membership levels considered and the trade-off rates between the membership functions are $\mu_1 - \mu_2$ = 9,001,242, $\mu_1 - \mu_3$ = 3.09E+08, $\mu_1 - \mu_4$ = 9,001,242, $\mu_1 - \mu_5$ = 9,001,242, $\mu_2 - \mu_3$ = 3.00E+08 $\mu_2 - \mu_4$ = 1.74E–03, $\mu_2 - \mu_5$ = 1.18E–02, $\mu_3 - \mu_4$ = 3.00E+08, $\mu_3 - \mu_5$ = 3.00E+08, $\mu_4 - \mu_5$ = 1.01E–02, $\mu_1 - \mu_2 - \mu_3$ = 3.09E+08, $\mu_1 - \mu_2 - \mu_4$ = 9,001,242, $\mu_1 - \mu_2 - \mu_5$ = 9,001,242, $\mu_1 - \mu_3 - \mu_4$ = 3.09E+08, $\mu_1 - \mu_3 - \mu_5$ = 3.09E+08, $\mu_1 - \mu_4 - \mu_5$ = 9,001,242, $\mu_2 - \mu_3 - \mu_4$ = 3.00E+08, $\mu_2 - \mu_3 - \mu_5$ = 3.00E+08, $\mu_2 - \mu_4 - \mu_5$ 1.18E–02, $\mu_3 - \mu_4 - \mu_5$ = 3.00E+08, $\mu_1 - \mu_2 - \mu_3 - \mu_4$ = 3.09E+08, $\mu_1 - \mu_2 - \mu_3 - \mu_5$ = 3.09E+08, $\mu_2 - \mu_3 - \mu_4 - \mu_5$ = 3.09E+08, $\mu_1 - \mu_2 - \mu_3 - \mu_4 - \mu_5$ = 3.09E+08. From these results, we can sort the objective functions efficiency degrees based interrelation levels like this $\mu_3(Z_3) > \mu_1(Z_1) > \mu_2(Z_2) = \mu_5(Z_5) > \mu_4(Z_4)$.

5. Conclusion

This study proposed a fuzzy mixed goal programming model addressing health care organization's resource allocation problem in a fuzzy environment. Compared to other deterministic techniques, the formulation can effectively handle the vagueness and imprecision in the statement of the objectives and ensure that the more importance of a fuzzy goal, the higher achievement degree it can obtain. Further, the formulation can easily be extended to other service organizations when the decision variables are vague and decision makers need to determine a desired achievement degree and preemptive priority for each of the fuzzy goals based on the relative importance of the goals. An example case with realistic data from the Turkeys health care organization structure showed the effectiveness and flexibility of our model to handle real world problems. With more information about the service structure and behaviors of healthcare organization, the system can set clear priority values as fuzzy weight (or importance) in future studies.

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