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ORIGINAL ARTICLE

Selecting and full ranking suppliers with imprecise data: A new DEA method

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Abstract Supplier selection, a multi-criteria decision making (MCDM) problem, is one of the most important strategic issues in supply chain management (SCM). A good solution to this problem significantly contributes to the overall supply chain performance. This paper proposes a new integrated mixed integer programming - data envelopment analysis (MIP-DEA) model for finding the most efficient suppliers in the presence of imprecise data. Using this model, a new method for full ranking of units is introduced. This method tackles some drawbacks of the previous methods and is computationally more efficient. The applicability of the proposed model is illustrated, and the results and performance are compared with the previous studies.

Keyword Data envelopment analysis · Imprecise data · Supplier selection · Full ranking method · Uncertainly · Best efficient unit

1 Introduction

Charnes et al. [6] introduced an innovative data oriented mathematical approach, data envelopment analysis (DEA), for evaluating numerous relative efficiency measurements of a set of peer decision making units (DMUs). Each DMU converts multiple inputs to multiple outputs. Numerous researches in various fields and various applications have quickly showed that DEA is an outstanding and straightforward methodology for modeling operational process in performance evaluations. As a result, this approach has rapidly gained too much attention and widespread use by the scientists. Nowadays, DEA becomes the important analysis tool and research method in management science, operational research, system engineering, decision analysis, and so on.

Generally, traditional DEA models partition all DMUs into two main groups: efficient and inefficient. These models fail to discriminate between efficient DMUs. To capture this shortcoming, many different approaches are proposed: Cross-efficiency methods utilize DEA in peer evaluation instead of only a self-evaluation [21]. Super-efficiency ranking methods eliminate the data on the under evaluation unit from the solution set [4]. Benchmark ranking methods rank efficient DMUs by measuring their importance as a benchmark for inefficient DMUs [32]. Ranking with multivariate statistics in DEA context, including linear discriminant analysis, uses the statistical techniques in alliance with DEA (Sinuany-Stern et al. [20]). In ranking inefficient DMUs approaches a new Measure of Inefficiency Dominance (MID) index is introduced to rank all inefficient units [5]. Notwithstanding with the multi-criteria decision making (MCDM) methodologies which do not consider a complete ranking as their ultimate aim, DEA-MCDM methods deal with the use of preference information to further improve the discriminating power of the DEA models [12].

Besides all ranking studies, the problem of finding a single efficient DMU, known as the most (best) efficient DMU in DEA, has called the attention of some researchers: Karsak and Ahiska [15] proposed a MCDM-DEA model in order to select the most efficient advanced manufacturing technology (AMT). Amin and Toloo [2] and Toloo and Nalchigar [24] tried to improve and extend the approach of Karsak and Ahiska [15]. Ertay et al. [9] suggested a minimax model in order to detect the most efficient layout in the facility layout design (FLD) problem. Farzipoor Saen [10] proposed an approach to deal with the supplier selection problem in supply chain system. Toloo et al. [25], Toloo and Nalchigar [27] and

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Amin et al. [3] formulated an integrated MIP model for finding the most efficient discovered rules in data mining and designed an algorithm to rank all efficient discovered rules. Farzipoor Saen [11] proposed an MIP model to handle the media selection problem. Some new approaches can be found in Toloo [31], Toloo and Ertay [29] and Toloo and Kresta [30]. Although ranking approaches can be applied to find the most efficient unit, it is unnecessary to rank all efficient DMUs and then find the most efficient one. More importantly, traditional DEA models usually consider variable set of optimal weights for each DMU, meanwhile the most efficient DMU should be founded in an identical condition which can be achieved by the common set of optimal weights (CSW) approaches. These approaches enjoy considerable advantages in terms computational issues. Instead of solving at least one optimization problem for each DMU in variable set of optimal weights approaches, an integrated model can be formulated to find the most efficient DMU in CSW approaches.

Supplier selection is a critical managerial decision making problem, which requires considering various qualitative and quantitative factors. The goal is to find the best supplier for meeting a firm's needs consistently and at an acceptable cost [16]. This problem is a fundamental issue of supply chain area, a kind of MCDM which requires MCDM methods for an effective problem solving. A good solution to this problem significantly contributes to the overall supply chain performance. Numerous methods have been proposed to tackle the problem of supplier selection. Analytic hierarchy process (AHP), artificial intelligence (AI), analytic network process (ANP), linear programming (LP), mathematical programming, multi-objective programming, DEA, neural networks (NN), and fuzzy set theory (FST) are instances of methods that have been proposed in literature. A comprehensive literature review of the MCDM approaches in supplier evaluation and selection is published by Ho et al. [14]. The readers are referred to this paper for further discussion on methods.

Recently, Toloo and Nalchigar [26] proposed a new DEA method for selecting and ranking suppliers in the presence of imprecise data, particularly when it is of cardinal, ordinal, or bounded form. As will be seen subsequently, their approach has some drawbacks. The aim of this study is to capture theses drawbacks. Toward this end, we propose a new integrated MIP-DEA model for finding the most efficient unit in the presence of cardinal and ordinal data. The model improves the integrated DEA model that was proposed in Toloo and Nalchigar [26]. In addition, using the suggested MIP-DEA model, we propose a new ranking method to prioritize units. This method is novel and computationally more efficient than the previous proposed methods.

The remainder of this paper is organized as follows: Section 2 reviews related studies and investigates the drawbacks in the previous works. Section 3 presents a new MIP-DEA model which tackles the drawbacks. A new method for full ranking units in the presence of cardinal and ordinal data is proposed in Section 4. Section 5 shows the applicability of proposed method and compares its results and performance with the previous methods. Finally, the paper closes in Section 6 with some concluding remarks.

2 Related works

Since supplier selection problem has been widely studied by many authors, various approaches have been proposed. For instance, Kilincci and Onal [16] proposed a fuzzy analytic hierarchy-based method to select the best supplier for a washing machine company in Turkey. Sevkli et al. [19] applied a hybrid method, data envelopment analytic hierarchy process (DEAHP) methodology to tackle the problem of supplier selection. Lin et al. [18] proposed a hybrid methodology of ANP technique for order preference by similarity to ideal solution (TOPSIS) and LP for supplier selection process. Güneri et al. [13] developed a new approach based on adaptive neuro-fuzzy inference system (ANFIS) to overcome supplier selection problem.

Although extensive research has been carried out on supplier selection problem, most of them focus on cardinal data. Hence, the proposed methods are not applicable for conditions in which data is imprecise, especially in the form of ordinal and interval data. In other words, in many real world supplier selection problems, the data of the alternatives is imprecise and many of the traditional methods are not applicable. To encounter these conditions, some authors have proposed supplier selections methods that deals with imprecise data. Among others, Farzipoor Saen [10] proposed the following model to identify a set of efficient suppliers in the presence of both cardinal and ordinal data, without considering the non-Archimedean epsilon to have positive decision variables.

$$
\begin{aligned}
&\pi_o^* = \max \sum_{r=1}^{s} Y_{ro} \\
&\text{s.t.} \\
&\sum_{i=1}^{m} X_{io} = 1 \\
&\sum_{r=1}^{s} Y_{rj} - \sum_{i=1}^{m} X_{ij} \leq 0 \qquad j = 1, 2, \ldots, n \\
&X_{ij} \in \widetilde{D}_i^- \qquad i = 1, 2, \ldots, m \qquad j = 1, 2, \ldots, n \\
&Y_{rj} \in \widetilde{D}_r^+ \qquad r = 1, 2, \ldots, s \qquad j = 1, 2, \ldots, n \\
&X_{ij} \geq 0 \qquad i = 1, 2, \ldots, m \qquad j = 1, 2, \ldots, n \\
&Y_{rj} \geq 0 \qquad r = 1, 2, \ldots, s \qquad j = 1, 2, \ldots, n
\end{aligned}
\tag{1}
$$

where $\widetilde{D}_i^-$ and $\widetilde{D}_r^+$ are the sets of constraints to consider:

- *Bounded data:* $\underline{y}_{rj}u_r \le Y_{rj} \le \overline{y}_{rj}u_r$, $\underline{x}_{ij}w_i \le X_{ij} \le \overline{x}_{ij}w_i$.
- *Weak ordinal data:* $Y_{rj} \le Y_{rk}$, $X_{ij} \le X_{ik}$.
- *Strong ordinal data:* $Y_{rj} < Y_{rk}$, $X_{ij} < X_{ik}$ $\forall j \ne k$ for some r, i.
- *Ratio bounded data:* $L_{rj} \le \frac{Y_{rj}}{Y_{rj_o}} \le U_{rj}$ and $G_{ij} \le \frac{X_{rj}}{X_{ij_o}} \le H_{ij}(j \ne j_o)$, where L_{rj} and G_{ij} represent the lower bounds, U_{rj} and H_{ij} represent the upper bounds.
- *Cardinal data:* $Y_{rj} = \mu_r \widehat{y}_{rj}$ and $X_{ij} = w_i \widehat{x}_{ij}$, where $\widehat{y}_{rj}$ and $\widehat{x}_{ij}$ represent cardinal data.

Given a set of DMUs, Model (1) finds a subset of efficient units and suggests them to decision maker. Although applicable in many situations, it has a main shortcoming since it identifies efficient suppliers and is not able to find the most efficient supplier candidates and rank them. In other words, using Farzipoor Saen [10]'s method, decision maker is not able to rank suppliers and chooses the best one. To overcome these drawbacks, Toloo and Nalchigar [26] proposed the following MIP model to select the most efficient supplier with the common set of weights:

$$
\begin{aligned}
& M^* = \min M \\
& \text{s.t.} \\
& M - d_j \ge 0 \qquad j = 1,2,\ldots,n \\
& \sum_{i=1}^{m} X_{ij} \le 1 \qquad j = 1,2,\ldots,n \\
& \sum_{r=1}^{s} Y_{rj} - \sum_{i=1}^{m} X_{ij} + d_j - \beta_j = 0 \qquad j = 1,2,\ldots,n \\
& \sum_{j=1}^{n} d_j = n-1 \qquad\qquad (2) \\
& 0 \le \beta_j \le 1 , d_j \in \{0,1\} \qquad j = 1,2,\ldots,n \\
& X_{ij} \in \widetilde{D}_i^- \qquad i = 1,2,\ldots,m \qquad j = 1,2,\ldots,n \\
& Y_{rj} \in \widetilde{D}_r^+ \qquad r = 1,2,\ldots,s \qquad j = 1,2,\ldots,n \\
& X_{ij} \ge \varepsilon^* \qquad i = 1,2,\ldots,m \\
& Y_{rj} \ge \varepsilon^* \qquad r = 1,2,\ldots,s \qquad j = 1,2,\ldots,n
\end{aligned}
$$

where ε^* is the non-Archimedean epsilon. According to Toloo and Nalchigar [26], the binary variable d_j is the deviation from the efficiency of DMU_j and hence their model minimizes the maximum of the deviations. Based on this assumption, they concluded that DMU_j is the most efficient unit if and only if $d_j^*=0$ and introduced an algorithm for ranking the suppliers. Nevertheless, based on the efficiency definition in DEA literature, DMU_j is CCR-efficient if and only if there exists at least one common set of optimal variables $(\mathbf{X}^*,\mathbf{Y}^*) \in R^{n(m+s)}$ such that $\sum_{r=1}^{s} Y_{rj}^* = \sum_{i=1}^{m} X_{ij}^*$. Considering the third types of constraints in Model (2), we can infer that DMU_j is a CCR-efficient DMU if and only if $d_j^* - \beta_j^* = 0$ and hence $d_j^* - \beta_j^*$ is the deviation from the efficiency of DMU_j. Therefore, this unit is the most efficient if and only if $d_j^* = \beta_j^*$. On the other hand, $-1 \le d_j - \beta_j \le 1$ and obviously it could not be considered as the deviation from the efficiency. In addition, it should be mentioned that a binary variable cannot be considered as deviation from efficiency, and this is the main drawback of Model (2). Considering these modeling drawbacks, the solution of Model (2) does not necessarily shows the most efficient unit. Moreover, Toloo and Nalchigar [26]'s algorithm for ranking units is based on Model (2); consequently, it suffers from the same drawbacks. In the next section, we propose a new MIP-DEA model to find the most efficient unit in the presence of imprecise data. In addition, a new ranking method is designed.

3 Proposed model

Toloo [23] proposed the following basic integrated LP model for finding a set of candidate DMUs for being the most BCC-efficient unit:

$$
\begin{aligned}
& z^* = \min d_{\max} \\
& \text{s.t.} \\
& d_{\max} - d_j \ge 0 \qquad j = 1,2,\ldots,n \\
& \sum_{i=1}^{m} w_i x_{ij} \le 1 \qquad j = 1,2,\ldots,n \\
& \sum_{r=1}^{s} u_r y_{rj} - u_0 - \sum_{i=1}^{m} w_i x_{ij} + d_j = 0 \qquad j = 1,2,\ldots,n \qquad\qquad (3) \\
& d_j \ge 0 \qquad j = 1,2,\ldots,n \\
& w_i \ge \varepsilon^* \qquad i = 1,2,\ldots,m \\
& u_r \ge \varepsilon^* \qquad r = 1,2,\ldots,s
\end{aligned}
$$

Solving this model, the user achieves a common set of strictly positive optimal weights $(\mathbf{u}^* > \mathbf{0}, \mathbf{w}^* > \mathbf{0})$, which gives us a set of the most BCC-efficient unit candidates. In other words, DMU_k is a candidate for the most BCC-efficient unit if and only if $\mathbf{u}^* \mathbf{y}_k - u_0 - \mathbf{w}^* \mathbf{x}_k = 0$ and hence in this model positive variable d_k^* is the deviation from efficiency of DMU_k. Indeed, DMU_k is a candidate for the most BCC-efficient unit if and only if $d_k^* = 0$.

In addition, Toloo [23] proposed the following integrated MIP-DEA model to find the most efficient unit from the set of candidates:

$$
\begin{aligned}
& z^* = \min d_{\max} \\
& s.t. \\
& d_{\max} - d_j \geq 0 && j = 1,2,\ldots,n \\
& \sum_{i=1}^{m} w_i x_{ij} \leq 1 && j = 1,2,\ldots,n \\
& \sum_{r=1}^{s} u_r y_{rj} - u_0 - \sum_{i=1}^{m} w_i x_{ij} + d_j = 0 && j = 1,2,\ldots,n \\
& \sum_{j=1}^{n} \theta_j = n-1 \\
& d_j \leq M\theta_j && j = 1,2,\ldots,n \\
& \theta_j \leq N d_j && j = 1,2,\ldots,n \\
& d_j \geq 0 && j = 1,2,\ldots,n \\
& \theta_j \in \{0,1\} && j = 1,2,\ldots,n \\
& w_i \geq \varepsilon^* && i = 1,2,\ldots,m \\
& u_r \geq \varepsilon^* && r = 1,2,\ldots,s
\end{aligned}
\tag{4}
$$

where M and N are large enough numbers. Obviously, Models (3) and (4) make an assumption that input and output data are exact numbers on a ratio scale. Hence, they are not applicable for situations in which data are imprecise. The term 'imprecise data' reflects the situations where some of the input and output data are only known to lie within bounded intervals (interval numbers) while other data are known only up to an order Despotis and Smirlis [8]. Cooper et al. [7] and Kim et al. [17] classified imprecise data into four main groups as bounded data, weak ordinal data, strong ordinal data, and ratio bounded data as follows:

Bounded data:

$$\underline{y}_{rj} \leq y_{rj} \leq \overline{y}_{rj} \quad \text{and} \quad \underline{x}_{ij} \leq x_{ij} \leq \overline{x}_{ij} \qquad \text{for } r \in \mathrm{BO}, i \in \mathrm{BI} \tag{5}$$

where $\underline{y}_{rj}$ and $\overline{y}_{rj}$ are the lower and the upper bounds for r^{th} bounded output, $\underline{x}_{ij}$ and $\overline{x}_{ij}$ are the lower and the upper bounds for i^{th} bounded inputs, and BO and BI represent the associated sets involving bounded outputs and inputs, respectively.

Weak ordinal data:

$$y_{rj} \leq y_{rk} \quad \text{and} \quad x_{ij} \leq x_{ik} \qquad \text{for } r \in \mathrm{DO}, i \in \mathrm{DI}$$

or,

$$y_{r1} \leq y_{r2} \leq \ldots \leq y_{rk} \leq \ldots \leq y_{rn} \;\; (r \in \mathrm{DO}), \tag{6}$$

$$x_{i1} \leq x_{i2} \leq \ldots \leq x_{ik} \leq \ldots \leq x_{in} \;\; (i \in \mathrm{DI}), \tag{7}$$

where DO and DI represent the sets of weak ordinal outputs and inputs, respectively.

Strong ordinal data:

$$y_{rj} < y_{rk} \quad \text{and} \quad x_{ij} < x_{ik} \qquad \text{for } j \neq k, r \in \mathrm{DO}, i \in \mathrm{DI}$$

or,

$$y_{r1} < y_{r2} < \ldots < y_{rk} < \ldots < y_{rn} \;\; (r \in \mathrm{SO}), \tag{8}$$

$$x_{i1} < x_{i2} < \ldots < x_{ik} < \ldots < x_{in} \;\; (i \in \mathrm{SI}), \tag{9}$$

where SO and SI represent the sets of strong ordinal outputs and inputs, respectively. Clearly, SO⊆DO and SI⊆DI.

Ratio bounded data:

$$L_{rj} \leq \frac{y_{rj}}{y_{rj_o}} \leq U_{rj} \qquad (j \neq j_o) \;\; (r \in \mathrm{RO}) \tag{10}$$

$$G_{ij} \leq \frac{x_{rj}}{x_{ij_o}} \leq H_{ij} \qquad (j \neq j_o) \;\; (i \in \mathrm{RI}) \tag{11}$$

where RO and RI represent the sets of ratio bounded outputs and inputs, respectively.

If we incorporate Eqs. (5)–(11) to Model (3) and transform it to a linear model (by adopting similar approach to Toloo and Nalchigar [26]), we get:

$$
\begin{aligned}
& z^* = \min d_{\max} \\
& s.t. \\
& d_{\max} - d_j \geq 0 && j = 1,2,\ldots,n \\
& \sum_{i=1}^{m} X_{ij} \leq 1 && j = 1,2,\ldots,n \\
& \sum_{r=1}^{s} Y_{rj} - \sum_{i=1}^{m} X_{ij} + d_j = 0 && j = 1,2,\ldots,n \\
& d_j \geq 0 && j = 1,2,\ldots,n \\
& X_{ij} \in \widetilde{D}_i^{+} \quad i = 1,2,\ldots,m && j = 1,2,\ldots,n \\
& Y_{rj} \in \widetilde{D}_r^{-} \quad r = 1,2,\ldots,s && j = 1,2,\ldots,n \\
& X_{ij} \geq \varepsilon^* \quad i = 1,2,\ldots,m && j = 1,2,\ldots,n \\
& Y_{rj} \geq \varepsilon^* \quad r = 1,2,\ldots,s && j = 1,2,\ldots,n
\end{aligned}
\tag{12}
$$

Table 1 Data of 18 suppliers

Supplier no. (DMU)	Inputs		Output
	TC	SR[a]	NB
1	253	5	[50,65]
2	268	10	[60,70]
3	259	3	[40,50]
4	180	6	[100,160]
5	257	4	[45,55]
6	248	2	[85,115]
7	272	8	[70,95]
8	330	11	[100,180]
9	327	9	[90,120]
10	330	7	[50,80]
11	321	16	[250,300]
12	329	14	[100,150]
13	281	15	[80,120]
14	309	13	[200,350]
15	291	12	[40,55]
16	334	17	[75,85]
17	249	1	[90,180]
18	216	18	[90,150]

[a] Ranking such that 18≡highest rank, …, 1≡lowest rank ($x_{218}>x_{216}>\ldots>x_{217}$)

In which $\widetilde{D}_i^+$ and $\widetilde{D}_r^-$ are the same sets of constraints as in Model (1). It should be mentioned that Model (12) finds a set of candidates for the most CCR-efficient unit with a common set of optimal variables $(\mathbf{X}^*,\mathbf{Y}^*)\in R^{n(m+s)}$ in the presence of imprecise data. The model finds the efficient unit(s) and it is easy to verify that in this model d_j is the deviation from efficiency of DMU$_j$ and thus this unit is efficient if and only if $d_j^*=0$, or equivalently $\sum_{r=1}^{s} Y_{rj}^*=\sum_{i=1}^{m} X_{ij}^*$.

The interest significance of the following lemma is that it allows one to formulate a simpler and more practical model than Model (4).

Lemma 1 In Model (12), $\forall j\ d_j\leq 1$

Proof Form the constraints in Model (12), we have

$$d_j=\sum_{i=1}^{m} X_{ij}-\sum_{r=1}^{s} Y_{rj}\leq 1-\sum_{r=1}^{s} Y_{rj}\leq 1$$

which completes the proof.

Let $E=\{j|d_j^*=0\ j=1,\ldots,n\}$ results from solving Model (12). The following theorem indicates the relation between this model and Model (1) which is proposed in Farzipoor Saen [10].

Theorem 1 DMU$_{k\in E}$ is CCR-efficient.

Proof Let $k\in E$ and $(\mathbf{X}^*,\mathbf{Y}^*)\in R^{n(m+s)}$ be the set of common optimal variables in Model (12). Hence, $\sum_{r=1}^{s} Y_{rk}^*=\sum_{i=1}^{m} X_{ik}^*$. Obviously, $(\mathbf{X}^*,\mathbf{Y}^*)$ is a feasible solution of Model (1) and the related objective function value is equal to one. Therefore, DMU$_{k\in E}$ is CCR-efficient.

After solving Model (12), two mutually exclusive alternatives can be occurred for E: If $k\in E$, then DMU$_k$ is definitely the best efficient DMU with the common set of optimal variables, $(\mathbf{X}^*,\mathbf{Y}^*)$. In this case, Model (12) *individually* is enough to determine the best efficient unit. Otherwise, this model fails to find a single efficient unit. To resolve this problem, we suggest adding some appropriate constraints to the model to impose it to find just a single efficient unit. We formulate the following MIP model to be applied in this situation:

$$
\begin{aligned}
& z^* = \min d_{\max} \\
& \text{s.t.} \\
& d_{\max}-d_j\geq 0 && j=1,2,\ldots,n \\
& \sum_{i=1}^{m} X_{ij}\leq 1 && j=1,2,\ldots,n \\
& \sum_{r=1}^{s} Y_{rj}-\sum_{i=1}^{m} X_{ij}+d_j=0 && j=1,2,\ldots,n \\
& \sum_{j=1}^{n}\theta_j=n-1 \\
& d_j\leq\theta_j && j=1,2,\ldots,n \\
& \theta_j\leq N d_j && j=1,2,\ldots,n \\
& X_{ij}\in\widetilde{D}_i^+ \quad i=1,2,\ldots,m && j=1,2,\ldots,n \\
& Y_{rj}\in\widetilde{D}_r^- \quad r=1,2,\ldots,s && j=1,2,\ldots,n \\
& \theta_j\in\{0,1\} && j=1,2,\ldots,n \\
& X_{ij}\geq\varepsilon^* \quad i=1,2,\ldots,m && j=1,2,\ldots,n \\
& Y_{rj}\geq\varepsilon^* \quad r=1,2,\ldots,s && j=1,2,\ldots,n
\end{aligned}
\tag{13}
$$

According to the model, there is only one zero auxiliary binary variable, says θ_k, and the constraint $d_k\leq\theta_k$ implies $d_k=0$. In this case, the constraint $\theta_k\leq N d_k$ is redundant. On the other hand, the constraint $\theta_j\leq N d_j$, for positive binary variables ($\theta_j=1$) and a large enough value for N, leads to positive d_j. Now, Lemma 1 insures that the constraint $d_j\leq\theta_j$ is redundant. Hence, the following Lemma is proved.

Lemma 2 In Model (13), there is only one unit with zero deviation.

It should be notice that Model (13) involves less parameter than Model (4) and this improvement is due to Lemma 1. As a result, Model (13) is simpler and more practical than Model (4). In a similar manner as in Toloo [31], a model for obtaining a suitable epsilon value in Model (13) can be extended.

In the next section, we utilize Models (12) and (13) to propose an innovative method for full ranking of units.

4 Ranking method

Various authors have studied the problem of ranking DMUs in DEA. Adler et al. [1] reviewed the ranking methods in DEA and categorized them into six groups. Interested readers are referred to this paper for further discussion on ranking methods. Although there are a lot of ranking methods in DEA, most of them focus on precise data. In other words, ranking units with imprecise data have not well studied. To fill this gap, in this section, we propose a method to rank DMUs as follows:

Step 0 Let $i=1$. Replace the constraint $\sum_{j=1}^{n} \theta_j = n-1$ in Model (13) with $\sum_{j=1}^{n} \theta_j = n-i$.

Step 1 Given all DMUs, solve Model (12). Let E be the set of candidates for the most efficient DMU. For each $\text{DMU}_{j\notin E}$, d_j^* is the deviation from the efficiency and could be used to rank $\text{DMU}_{j\notin E}$. DMU_u has a better rank than DMU_v if and only if $d_u^*<d_v^*$.

Step 2 If $|E|=1$, then stop. $\text{DMU}_{k\in E}$ is the most efficient unit and ranked first.

Step 3 Given all DMUs, solve Model (13) and suppose $d_k^*=0$. DMU_k gets the i^{th} rank position.

Step 4 If for all distinct $u,v\in E$ we have $d_u^*\neq d_v^*$, then stop. DMU_u has a better rank than DMU_v if and only if $d_u^*<d_v^*$.

Step 5 Let $i=i+1$.

Step 6 If $i=|E|$, then stop. Otherwise, go to Step 3.

Indeed, in Step 1 of the proposed method, a set of DMUs is selected as candidates to be the most efficient unit. In this step, those DMUs, which are marked as non-candidate, are ranked based on their deviation from the efficiency score (d_j^*). In Step 2, if we have only one candidate to be the most efficient unit, we stop the algorithm since all DMUs are ranked. In this case, we have $\sum_{r=1}^{s} Y_{rk}^*=\sum_{i=1}^{m} X_{ik}^*$ and $\forall j\neq k$ $\sum_{r=1}^{s} Y_{rj}^*-\sum_{i=1}^{m} X_{ij}^*<0$ and clearly DMU_k is the most efficient unit. If this not were the case, then Model (13) finds the most efficient unit and this unit gets the first rank position. To rank other candidates for the most efficient unit, we must consider their deviation from the efficiency. If all distinct candidates have different deviation from the efficiency, then these units can be ranked based on their deviation value. Otherwise, Model (13) with new updated constraint ($\sum_{j=1}^{n} \theta_j=n-i$) finds the next top ranked DMU. In other words, if $i=t$, then this algorithm rank t^{th} most efficient candidate. As a result, the designated algorithm must be repeated at most $|E|-1$ times to rank all the candidates.

The next section of this paper shows the applicability of proposed method and compares its result with previous methods.

Table 2 Results of the proposed method

Supplier no. (DMUs)	Ranking results from Model (12)	$i=1$ Ranking results from Model (13)	$i=2$ Ranking results from Model (13)
1	8	8	8
2	14	14	14
3	13	13	13
4	4	4	4
5	12	12	12
6	5	5	5
7	6	6	6
8	–	–	2
9	9	9	9
10	10	10	10
11	–	1	1
12	15	15	15
13	16	16	16
14	7	7	7
15	17	17	17
16	18	18	18
17	–	–	3
18	11	11	11

5 Application

In this section, the application of the proposed method and models is shown in a supplier selection problem previously used by Toloo and Nalchigar [26]. Including 18 suppliers, data

Table 3 Comparison of the proposed method with the previous methods

Supplier	Results of Toloo and Nalchigar [26]	Results of the Proposed Method	Supplier status
DMU_4	1	4	Non-candidate
DMU_{14}	2	7	Non-candidate
DMU_6	3	5	Non-candidate
DMU_{17}	4	3	Candidate
DMU_{11}	5	1	Candidate
DMU_8	6	2	Candidate
DMU_9	7	9	Non-candidate
DMU_7	–	6	Non-candidate

is obtained from other previous studies that used DEA for problem of supplier selection [10, 22]. These suppliers consume two inputs as total cost of shipment (TC) and supplier reputation (SR) to produce bills received from the supplier without errors (NB) as output. The inputs are in cardinal and ordinal scale, respectively, and the output is bounded data. Table 1 presents the data.

By adopting Zhu [33]'s approach, inputs and outputs of suppliers could be written as follows:

$$\widetilde{D}_1^- = \{X_{11} = 253w_1; X_{12} = 268w_1; X_{13} = 259w_1, \ldots, X_{118} = 216w_1\}$$

$$\widetilde{D}_2^- = \{X_{218} \geq X_{216} \geq \ldots \geq X_{217}\}$$

$$\widetilde{D}_1^+ = \{50\mu_1 \leq Y_{11} \leq 65\mu_1; 60\mu_1 \leq Y_{12} \leq 70\mu_1; 40\mu_1 \leq Y_{13} \leq 50\mu_1; \ldots; 90\mu_1 \leq Y_{118} \leq 150\mu_1\}$$

Using these relations, we solve Model (12). The results indicate that $E = \{8, 11, 17\}$ which means DMU_8, DMU_{11}, and DMU_{17} are the candidates to be the most efficient DMU. For the rest of DMUs, $d^*_{j \notin E}$ is the deviation from the efficiency score and is used to rank them (See Table 2). Notice that based on the results of Model (1) suppliers 4, 6, 8, 9, 11, 14, and 17 are recognized as the efficient suppliers and as we expected the most efficient supplier candidates selected among these efficient suppliers.

Since the condition of Step 2 is not satisfied, we skip this step and directly proceed to Step 3. In this step, we solve Model (13) with i=1. The result shows that d^*_{11}=0 and d^*_8= d^*_{17}=0.001; hence, we skip Step 4. In Step 5, since i=2<|E|=3, we go to Step 3 and solve Model (13) with i=2. The result indicates that d^*_8=0. As a result, DMU_8 gets the second rank position. Obviously, the last candidate, DMU_{17} , is ranked third. The stop condition in Step 5 is met and all DMUs are ranked. Table 2 presents the full ranking results.

To provide further insights, we compare the performance and results of proposed method with the results of two previous methods on the same data set (See Table 3).

The last column in Table 3 indicates that either a DMU is selected as the most efficient unit candidate (candidate) or not (non-candidate). This table illustrates the main drawback of Toloo and Nalchigar [26]'s method. The first three top ranked DMUs in this method (i.e., DMU_4, DMU_{14}, and DMU_6) are not *even* identified as the most efficient unit candidates. Apparently, a reliable approach is required to rank top DMUs among the most efficient unit candidates (i.e., DMU_{11}, DMU_8, and DMU_{17}). This inconsistency in the method, as explained in Section 2, lies in the fact that Model (2) is formulated incorrectly. As can be extracted from Table 3, notwithstanding Toloo and Nalchigar [26]'s method, the proposed method in this paper results in reliable ranking. Indeed, the most efficient unit candidates are at the top of ranking in our new suggested method. Hence, in comparison with the previous method, the proposed method provides more decision aids to decision maker and captures drawbacks of the previous studies.

Finally, to compare these methods from computational efficiency, we can say that Farzipoor Saen [10]'s method requires decision maker to solve 18 LPs to find 7 efficient suppliers whereas in our approach, 2 (=|E|−1) MIPs must be solved. Although dealing with LPs is more computationally efficient than MIPs, Farzipoor Saen [10]'s method selects suppliers based on variable set of optimal weights meanwhile our integrated approach is based on the common set of optimal weights. Common set of optimal weights lets us to select suppliers in an identical condition. As a result, the suggested method in this study is more logical and practical.

6 Conclusion

Supplier selection has been considered as one of the most important strategic issues in SCM. Many practitioners and researchers have emphasized that a well designed and implemented supply chain system play a critical role in increasing competitive advantages of companies. This paper developed a supplier selection approach based on DEA for conditions in which data of suppliers are imprecise. Although previous researches have considered the problem of imprecise data in supplier selections, their approach suffers from some drawbacks. The main contribution of this paper was to propose a new integrated MIP-DEA model to overcome these drawbacks and to use the new model to propose a new method for full ranking of units. The applicability of the new method was shown, and results were compared with the previous studies. Future research could use the proposed method of this paper, with mirror modifications, for other multi-criteria managerial decision making problems such as selection of media, technology, and international market. Moreover, the formulated models and designated algorithm in this study can be extended to find the most BCC-efficient unit in the presence of imprecise data.

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References

1. Adler N, Friedman L, Stern ZS (2002) Review of ranking methods in the data envelopment analysis context. Eur J Oper Res 140:249–265
2. Amin GR, Toloo M (2007) Finding the most efficient DMUs in DEA: an improved integrated model. Comput Ind Eng 52(2):71–77
3. Amin GR, Gattoufi S, Rezaee Seraji E (2011) A maximum discrimination DEA method for ranking association rules in data mining. Int J Comput Math 88(11):2233–2245.
4. Andersen P, Petersen NC (1993) A procedure for ranking efficient units in data envelopment analysis. Manag Sci 39(10):1261–1294
5. Bardhan I, Bowlin WF, Cooper WW, Sueyoshi T (1996) Models for efficiency dominance in data envelopment analysis. Part I: Additive models and MED measures. J Oper Res Soc Jpn 39:322–332
6. Charnes A, Cooper WW, Rhodes E (1978) Measuring the efficiency of decision-making units. Eur J Oper Res 2:429–444
7. Cooper WW, Park KS, Yu G (1999) IDEA and AR-IDEA: Models for dealing with imprecise data in DEA. Manag Sci 45:597–607
8. Despotis DK, Smirlis YG (2002) Data envelopment analysis with imprecise data. Eur J Oper Res 140(1):24–36
9. Ertay T, Ruan D, Tuzkaya UR (2006) Integrating data envelopment analysis and analytic hierarchy for the facility layout design in manufacturing systems. Inf Sci 176:237–262
10. Farzipoor Saen R (2007) Suppliers selection in the presence of both cardinal and ordinal data. Eur J Oper Res 183:741–747
11. Farzipoor Saen R (2011) Media selection in the presence of flexible factors and imprecise data. J Oper Res Soc 62:1695–1703
12. Golany B (1988) An interactive MOLP procedure for the extension of data envelopment analysis to effectiveness analysis. J Oper Res Soc 39(8):725–734
13. Güneri AF, Ertay T, Yücel A (2011) An approach based on ANFIS input selection and modeling for supplier selection problem. Expert Syst Appl 38(12):14907–14917
14. Ho W, Xu X, Dey PK (2010) Multi-criteria decision making approaches for supplier evaluation and selection: a literature review. Eur J Oper Res 202(1):16–24
15. Karsak EE, Ahiska SS (2005) Practical common weight multi-criteria decision-making approach with an improved discriminating power for technology selection. Int J Prod Res 43(8):1537–1554
16. Kilincci O, Onal SA (2011) Fuzzy AHP approach for supplier selection in a washing machine company. Expert Syst Appl 38(8):9656–9664
17. Kim SH, Park CG, Park KS (1999) An application of data envelopment analysis in telephone offices evaluation with partial data. Comput Oper Res 26:59–72
18. Lin CT, Chen CB, Ting YC (2011) An ERP model for supplier selection in electronics industry. Expert Syst Appl 38(3):1760–1765
19. Sevkli M, Koh SCL, Zaim S, Demirbag M, Tatoglu E (2007) An application of data envelopment analytic hierarchy process for supplier selection: a case study of BEKO in Turkey. Int J Prod Res 45(9):1973–2003
20. Sinuany-Stern Z, Mehrez A, Barboy A (1994) Academic department efficiency via data envelopment analysis. Comput Oper Res 21(5):543–556
21. Sexton TR, Silkman RH, Hogan AJ (1986) Data envelopment analysis: Critique and extensions. In: Silkman RH (ed) Measuring efficiency: an assessment of data envelopment analysis. Jossey-Bass, San Francisco, pp 73–105
22. Talluri S, Baker RC (2002) A multi-phase mathematical programming approach for effective supply chain design. Eur J Oper Res 141:544–558
23. Toloo M (2012) On finding the most BCC-efficient DMU: a new integrated MIP-DEA model. Appl Math Model 36:5515–5520
24. Toloo M, Nalchigar S (2009) A new integrated DEA model for finding most BCC-efficient DMU. Appl Math Model 33:597–604
25. Toloo M, Sohrabi B, Nalchigar S (2009) A new method for ranking discovered rules from data mining by DEA. Expert Syst Appl 36:8503–8508
26. Toloo M, Nalchigar S (2011) A new DEA method for supplier selection in presence of both cardinal and ordinal data. Expert Syst Appl 38:14726–14731
27. Toloo M, Nalchigar S (2011) On ranking discovered rules of data mining by data envelopment analysis: some models with wider applications. In: Funatsu K, Hasegawa K (Eds) New fundamental technologies in data mining. InTech publisher 425–446. (Online available at: http://www.intechweb.org/books/show/title/new-fundamental-technologies-in-datamining)
28. Toloo M (2014) An epsilon-free approach for finding the most efficient unit in DEA. Appl Math Model 38:3182–3192.
29. Toloo M, Ertay T (2014) The most cost efficient automotive vendor with price uncertainty: a new DEA approach. Meas 52:135–144.
30. Toloo M, Kresta A (2014) Finding the best asset financing alternative: a DEA-WEO approach. Meas. DOI: http://dx.doi.org/10.1016/j.measurement.2014.05.015
31. Toloo M (2014) The role of non-Archimedean epsilon in finding the most efficient unit: with an application of professional tennis players. Appl Math Modell. http://dx.doi.org/10.1016/j.apm.2014.04.010
32. Torgersen AM, Forsund FR, Kittelsen SAC (1996) Slack-adjusted efficiency measures and ranking of efficient units. J Prod Anal 7:379–398
33. Zhu J (2003) Imprecise data envelopment analysis (IDEA): a review and improvement with an application. Eur J Oper Res 144:513–529