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Robust decoupling control synthesis

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***Abstract*— The control of multivariable industrial processes is frequently performed using a multiloop configuration, in which several PID controllers are committed to control different channels of the plant. A difficulty with such a strategy arises due to the interaction among the control loops, which may cause the control action in a loop to give rise to significant disturbances in other loops. In some cases, it is mandatory to consider a decoupling control that includes a decoupling precompensator, or decoupler, to guarantee acceptable decoupling among the control loops. This paper presents a new robust decoupling control synthesis procedure for multiloop control systems which aims to decouple the different channels of the multivariable system and to guarantee the tracking response performance. The control problem is stated as a non-convex optimization problem which is formulated directly in the space of the PI/PID controllers and precompensator parameters. Polytopic models represent the system uncertainty. An application example is developed for the control of a quadruple-tank process with emphasis in dealing with the control decoupling when the system is working on a non-minimum phase operating point.**

***Index Terms*— Decoupling control, multiloop control, PID control, polytopic uncertainty.**

I. Introduction

The usage of PI/PID controllers for the implementation of multiloop control for multivariable processes is quite popular in industry because such controllers are easy to understand, implement and tune by operators, and decentralized structures are failure-tolerant [1]. In the multiloop control architecture, each manipulated variable depends only on a single controlled variable. If process interactions are significant, when a manipulated variable strongly affects more than one controlled variable, even the best multiloop control system may not provide satisfactory control. The decoupling control scheme is one of the early approaches to deal with undesirable closed loop interactions. This control scheme combines the multiloop PI/PID control configuration with a static or dynamic decoupling precompensator, or decoupler, in order to compensate for the plant interactions, as illustrated in Fig. 1. The drawback of the increased complexity of the control architecture is justified by the improved decoupling among control loops and better tracking response performance. The task of developing a satisfactory decoupling control in multiloop control architectures constitutes a problem that has received great interest in the last decades (see [2], [3], [4], [5] and references therein). Most of the decoupling control synthesis strategies firstly compute the precompensator to turn the resultant system into a more nearly diagonal transfer matrix and them compute the multiloop PI/PID controllers. Dynamic decoupling is often very difficult due to the presence of uncertainty in the plant model and due to the lack of realizability of "ideal" decouplers. In conventional decoupler synthesis based on the inverse of the plant transfer matrix, when the system presents non-minimum phase elements or relative degree one or more, the resulting decoupler can become non-causal or unstable.

The contribution of this paper is to present a robust decoupling control synthesis procedure that aims to decouple the control channels of uncertain discrete-time linear time-invariant multivariable systems assuring the tracking response performance. In this paper, the PID tuning procedure presented in [6] is extended to the decoupling control synthesis problem. Differently from conventional approaches, the proposed synthesis procedure computes the multiloop PI/PID controllers and the decoupling precompensator simultaneously based on a reference model approximation scheme. It is considered a block diagonal reference model to guarantee the decoupling and tracking response performance [7], [8]. The advantages of the proposed synthesis procedure is to consider the model uncertainty and to guarantee that the suboptimal decoupling precompensator is realizable. The decoupling control synthesis is formulated as an

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optimization problem considering state space model and polytopic uncertainty.

Similar formulations, also based on a two-step iterative procedure that alternates an analysis step and a non-linear optimization synthesis step which is performed directly in the space of controller variables, have been proposed for different control problems, achieving high-performance synthesis results [9], [10], [6], [7], [8].

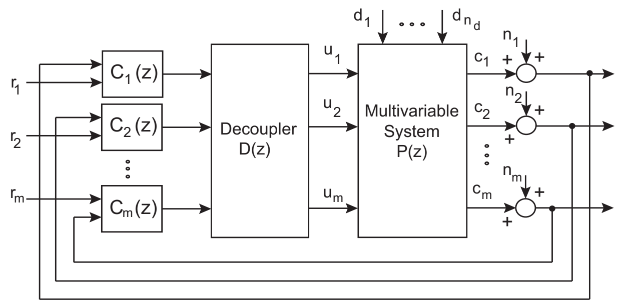

Fig. 1. Decoupling Control.

The notation in this paper is standard. The compact notation:

$$G(z) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

is applied to denote the transfer matrix $G(z) = C(zI - A)^{-1}B + D$ and $x(k) \triangleq x(kT)$, T the sampling time.

II. Problem Formulation

Consider a discrete-time linear time-invariant system described by

$$\begin{array}{rcl} x(k+1) & = & Ax(k) + B_u u(k) + B_w w(k), \\ z(k) & = & C_z x(k) + D_{zu} u(k) + D_{zw} w(k), \\ y(k) & = & C_y x(k) + D_{yw} w(k), \end{array} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^m$ is the control signal vector (manipulated variables), $w(k) \in \mathbb{R}^{n_w}$ is the exogenous input vector, $z(k) \in \mathbb{R}^{n_z}$ is the controlled output vector, and $y(k) \in \mathbb{R}^{n_y}$ is the measured output vector (inputs to the dynamic output-feedback controller).

To simplify the notation, the system matrices in Eq. (1) are gathered in the matrix:

$$P \triangleq \left[\begin{array}{c|cc} A & B_u & B_w \\ \hline C_z & D_{zu} & D_{zw} \\ C_y & 0 & D_{yw} \end{array}\right], \tag{2}$$

that can include uncertain parameters belonging to a known convex compact set, or polytope, defined by its vertices:

$$\mathcal{P}(\alpha) \triangleq \left\{ P : P = \sum_{i=1}^{N} \alpha_i P_i; \quad \alpha \in \Omega \right\}, \tag{3}$$

$$\Omega \triangleq \left\{ \alpha \ : \ \alpha_i \geq 0, \ \sum_{i=1}^{N} \alpha_i = 1 \right\}, \tag{4}$$

with P_i, $i = 1, \ldots, N$, the polytope vertices and $\alpha = \left[\begin{array}{ccc} \alpha_1 & \ldots & \alpha_N \end{array} \right]'$ the vector that parameterizes the polytope. The dependence of the system matrices with α will be omitted.

Let the decentralized PI controller be represented by

$$C(z) \triangleq \left[\begin{array}{c|c} A_c & B_c \\ \hline C_c & D_c \end{array}\right],$$

and the decoupling precompensator be represented by

$$D(z) \triangleq \left[\begin{array}{c|c} A_d & B_d \\ \hline C_d & D_d \end{array}\right].$$

The dynamic output-feedback control, $U(z) = K(z)Y(z)$, is the product of these 2 blocks, $K(z) = D(z)C(z)$:

$$K(z) \triangleq \left[\begin{array}{c|c} A_k & B_k \\ \hline C_k & D_k \end{array}\right] = \left[\begin{array}{cc|c} A_d & B_d C_c & B_d D_c \\ 0 & A_c & B_c \\ \hline C_d & D_d C_c & D_d D_c \end{array}\right].$$

The closed-loop transfer matrix relating the controlled variables, $z(k)$, and the exogenous inputs, $w(k)$,

$$T_{zw}(z) = \left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right], \tag{5}$$

can be computed by

$$\begin{array}{l} A_f = \left[\begin{array}{cc} A + B_u D_k C_y & B_u C_k \\ B_k C_y & A_k \end{array} \right], \\ B_f = \left[\begin{array}{c} B_w + B_u D_k D_{yw} \\ B_k D_{yw} \end{array} \right], \\ C_f = \left[\begin{array}{cc} C_z + D_{zu} D_k C_y & D_{zu} C_k \end{array} \right], \\ D_f = \left[\begin{array}{c} D_{zw} + D_{zu} D_k D_{yw} \end{array} \right]. \end{array} \tag{6}$$

Let

$$T_m(z) = \left[\begin{array}{c|c} A_m & B_m \\ \hline C_m & D_m \end{array}\right], \ T_{cr}(z) = \left[\begin{array}{c|c} A_{cr} & B_{cr} \\ \hline C_{cr} & D_{cr} \end{array}\right], \tag{7}$$

where $T_m(z)$ is a block diagonal reference model that decouples the system and attains the tracking transient response specifications (overshoot, settling time, etc.) for each controlled output:

$$T_m(z) = \left[\begin{array}{ccc} T_{m,1}(z) & & 0 \\ & \ddots & \\ 0 & & T_{m,m}(z) \end{array} \right], \tag{8}$$

and $T_{cr}(z)$ is the closed-loop transfer function relating the set-point signals and the plant outputs, one of the blocks of $T_{zw}(z)$. The approximation error between the reference model and the closed-loop transfer function, $E(z) \triangleq T_m(z) - T_{cr}(z)$, can be represented by the following state-space model:

$$E(z) = \left[\begin{array}{cc|c} A_m & 0 & B_m \\ 0 & A_{cr} & B_{cr} \\ \hline C_m & -C_{cr} & D_m - D_{cr} \end{array}\right]. \tag{9}$$

This paper will consider a robust control problem that can be stated as: given a polytope-bounded uncertain, discrete-time, linear time-invariant system, $\mathcal{P}(\alpha)$, $\alpha \in \Omega$, and a reference model, $T_m(z)$, find the decoupling controller, $K(z) = D(z)C(s)$, that minimizes the maximum $\mathcal{H}_\infty$-norm of the error between the reference model and the closed-loop transfer function, $E(z)$, in the uncertainty domain:

$$\begin{array}{l} K^* = \arg\min\limits_{K} \max\limits_{\alpha \in \Omega} \|E(z, \alpha, K)\|_\infty \\ \text{subject to: } \ K \in \mathcal{F}, \end{array} \tag{10}$$

with $\mathcal{F}$ the set of decoupling controllers with a specified structure such as the closed-loop system is robustly stable.

The idea behind the proposed decoupling control problem formulation is simple. With a diagonal reference model, if the worst case approximation error is low it means that the gain of the out-of-diagonal elements of the closed-loop transfer matrix, $T_{cr}(z)$, will be closer to zero for all frequencies, leading to an satisfactory decoupling among the control loops. Besides, the diagonal elements of $T_{cr}(z)$ will approximate the specified frequency response.

III. Proposed Robust Decoupling Control Synthesis Procedure

The proposed procedure to tackle the non-convex optimization problem (10) directly in the space of controller parameters is based on two steps: synthesis and analysis. In the synthesis step, it is applied a non-linear optimization algorithm to solve the optimization problem (10) with the infinite set Ω replaced by a finite set of points $\tilde{\Omega} \subset \Omega$. This finite set is initially the set of vertices of the polytope as considered in convex formulations. To consider only the polytope vertices is not sufficient to guarantee the robust stability of the closed-loop system and the minimization of $\|E\|_\infty$ for all $\alpha \in \Omega$. To verify the decoupling controller computed in the first step, in the second step, it is applied an analysis procedure based on a combination of a branch-and-bound algorithm and LMI formulations [11]. If the analysis procedure finds an instance of an unstable system in the uncertain domain or if it is verified that the maximum value of $\|E\|_\infty$ does not occur in a point belonging to $\tilde{\Omega}$, then this point is included in $\tilde{\Omega}$ and it is necessary to execute the two steps of the procedure again. The procedure ends when it is verified that the closed-loop system is robustly stable and the maximum value of the objective function occurs on a point that belongs to $\tilde{\Omega}$ (or near to that set, accordingly to a specified accuracy).

In the synthesis step, the scalar optimization problem can be solved by means of the cone-ellipsoidal algorithm [12]. Let $\chi \in \mathbb{R}^d$ be the vector of optimization parameters (in this case the PI controllers and decoupling precompensator parameters), $f(\chi) : \mathbb{R}^d \to \mathbb{R}$ be the objective function to be minimized, and $g_i(\chi) : \mathbb{R}^d \to \mathbb{R}$, $i = 1, \ldots, s$, be the set of constraint functions. Let χ_k be the ellipsoid center and $Q_k = Q_k^T \succ 0$ the matrix that determines the direction and dimension of the ellipsoid axes. Given the initial values χ_0 and Q_0, the ellipsoidal algorithm is described by the following recursive equations:

$$\begin{aligned} \chi_{k+1} &= \chi_k - \frac{1}{d+1} Q_k \tilde{m}, \\ Q_{k+1} &= \frac{d^2}{d^2-1}\left(Q_k - \frac{2}{d+1} Q_k \tilde{m}\tilde{m}^T Q_k\right), \end{aligned} \tag{11}$$

with

$$\tilde{m} = m_k / \sqrt{m_k^T Q_k m_k},$$

where m_k is the sum of the normalized gradients (or sub-gradients) of the violated constraint functions, $\nabla g_i(\chi) > 0$, when χ_k is not a feasible solution, or the gradient (or sub-gradient) of the objective function, $\nabla f(\chi)$, when χ_k is a feasible solution. The gradients (or sub-gradients) are computed numerically by means of the finite difference method. Let e^i be the i-th column of the identity matrix of size d, $I_{d\times d}$, and δ be a scalar such that $\delta > 0$ and $\delta \to 0$ (typical values are in the range from 10^{-8} to 10^{-3}). Each entry of the vector $\nabla f(\chi)$ can be computed as:

$$v_i = \frac{f(\chi + \delta e^i) - f(\chi)}{\delta}, \quad i = 1, \ldots, d$$

In the analysis step, it is required to compute the $\alpha \in \Omega$ corresponding to the maximum of the objective and constraint functions in (10) or to find an $\alpha \in \Omega$ that corresponds to an unstable system, if $K \notin \mathcal{F}$. The basic strategy of the branch-and-bound algorithm is to partition the uncertainty domain, Ω, such as lower and upper bound functions converge to the maximum value of the norm in the uncertain domain Ω. This algorithm ends when the difference between the bound functions is lower than the prescribed relative accuracy. The algorithm is implemented considering as lower bound function the $\mathcal{H}_\infty$ norm computed in the vertices and as upper bound function the $\mathcal{H}_\infty$ guaranteed cost computed by means of linear matrix inequality (LMI) formulations, both functions calculated for the original polytope and its subdivisions [11]. If the system is not robustly stable, the algorithm finds an unstable system in the polytope while searching for the maximum norm value. A partition technique based on simplicial meshes [13] is applied to allow this procedure to be applied to polytopic models with improved efficiency. The $\mathcal{H}_\infty$ guaranteed cost is computed based on the LMI formulation of [14, Theorem 2].

In most of the cases, the worst case of the objective function is over a polytope vertex and the proposed procedure requires just one iteration. An example of a problem that requires more than one iteration of the proposed procedure is presented in [10].

IV. Illustrative Example

The proposed robust decoupling control synthesis procedure is illustrated using a quadruple-tank process, presented in Fig. 2. This is a laboratory process with an adjustable zero, that has been used to illustrate many issues in multivariable control [15], [16]. The two lower tank levels are controlled by means of two pumps. The three-way valve settings establish the interaction between the two control loops.

Considering deviations around an operating point, with all inputs and outputs as voltage signals, the quadruple-tank process can be represented by the lin-

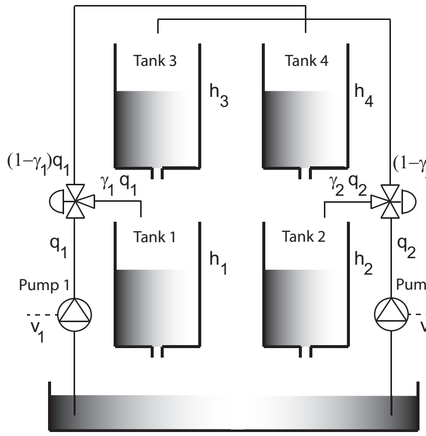

Fig. 2. Quadruple-tank process.

earized state space model [15]:

$$\begin{aligned}
\frac{dx}{dt} &= \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2 T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} x \\
&+ \begin{bmatrix} 0 & \frac{\gamma_1 k_1}{A_1} \\ \frac{\gamma_2 k_2}{A_2} & 0 \\ \frac{(1-\gamma_2)k_2}{A_3} & 0 \\ 0 & \frac{(1-\gamma_1)k_1}{A_4} \end{bmatrix} u, \qquad (12) \\
z &= \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} x, \\
y &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} w,
\end{aligned}$$

where the state variables are the four tank level deviations, $x_i = h_i - h_i^0$, $i = 1, 4$; the control signals are the two pump voltage deviations, $u \triangleq [v_2 - v_2^0 \; v_1 - v_1^0]^T$; the exogenous inputs are the reference signals, $w = [r_1 \; r_2]^T$; the controlled variables are the measured level signals of tanks 1 and 2, $z_j = k_c(h_j - h_j^0)$, $j = 1, 2$, k_c the sensor gain; the measured variables are the reference signals and the measured level signals, $y = [r_1 \; r_2 \; z_1 \; z_2]^T$. The time constants are

$$T_i \triangleq \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}, \quad i = 1, \ldots, 4. \qquad (13)$$

The quadruple-tank linearized model has the following parameter values [15]: tank cross-sections $A_1 = A_3 = 28\text{cm}^2$, $A_2 = A_4 = 32\text{cm}^2$; cross-section of the outlet holes $a_1 = a_3 = 0.071\text{cm}^2$, $a_2 = a_4 = 0.057\text{cm}^2$; sensor gain $k_c = 0.50\text{V/cm}$; and acceleration of gravity $g = 981\text{cm/s}^2$. In [15], two operating points are presented, P_- and P_+, which have minimum phase and non-minimum phase characteristics, respectively. The minimum phase operating point can be decoupled satisfactorily by means of decentralized PI controllers [8]. The non-minimum phase operating point is more difficult to decouple and it will be considered here. The corresponding parameter values of the non-minimum phase operating point is

TABLE I
PARAMETER VALUES OF THE NON-MINIMUM PHASE OPERATING POINT.

Parameter		Value
(h_1^0, h_2^0)	[cm]	(12,6; 13,0)
(h_3^0, h_4^0)	[cm]	(4,8; 4,9)
(v_1^0, v_2^0)	[V]	(3,15; 3,15)
(k_1, k_2)	[cm^3/Vs]	(3,14; 3,29)
(γ_1, γ_2)		(0,43; 0,34)
(T_1, T_2)		(63; 91)
(T_3, T_4)		(39; 56)

reproduced here in Table I. The tank inlet flows are function of the pump coefficients, k_1 and k_2, and the three-way valve coefficients, γ_1 and γ_2. In the case of the non-minimum phase operating point, it is better to apply pump 1 to control the level of the tank 2 and pump 2 to control the level of tank 1. The control signal vector is changed in relation to [15] to consider this better pairing.

It will be considered a variation of $\pm 10\%$ over the following uncertain parameters: T_1 and T_2. The system will be represented by a polytopic model with 4 vertices corresponding to the combination of the lower and upper values of the 2 uncertain parameters.

The choice of the reference model is based on a simple trial and error scheme to achieve a trade-off between decoupling and tracking response performance. The reference model is firstly chosen to achieve the desired tracking response performance. If the worst case of the approximation error, $\max_{\alpha \in \Omega} \|E(z)\|_\infty$, is high, resulting in an unsatisfactory decoupling, the reference model must be adjusted to reduce $\max_{\alpha \in \Omega} \|E(z)\|_\infty$, improving the decoupling. To adjust the reference model to reduce the approximation error, and consequently the coupling among control loops, it is necessary to choose its diagonal elements to reproduce the tracking transient responses that were already achieved with the previous reference model. In this specific case, it is necessary to include a first order Padé approximation of time delay in the final reference model to achieve a better trade-off:

$$T_{m,i}(s) = \frac{\omega_{n,i}^2(-\tau_i s + 1)}{(\tau_i s + 1)(s^2 + 2\zeta_i \omega_{n,i} s + \omega_{n,i}^2)}, \quad i = 1, 2. \qquad (14)$$

with $\tau_1 = 50$, $\omega_{n,1} = 0.01$, $\zeta_1 = 0.8$, $\tau_2 = 60$, $\omega_{n,2} = 0.01$, and $\zeta_2 = 1.5$. Considering sampling time $T_s = 2.5$s:

$$T_{m,1}(z) = \frac{-0.00029817(z - 1.051)(z + 0.9544)}{(z - 0.9512)(z^2 - 1.96z + 0.9608)}$$

$$T_{m,2}(z) = \frac{-0.00029638(z - 1.043)(z + 0.9486)}{(z - 0.9905)(z - 0.9592)(z - 0.9366)}$$

Consider a multiloop I-P controller (proportional action applied to output, $a = 0$) represented as

$$A_c = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B_c = \begin{bmatrix} \frac{T}{T_{i,1}} & 0 & -\frac{T}{T_{i,1}} & 0 \\ 0 & \frac{T}{T_{i,2}} & 0 & -\frac{T}{T_{i,2}} \end{bmatrix},$$

$$C_c = \begin{bmatrix} k_{p,1} & 0 \\ 0 & k_{p,2} \end{bmatrix},$$

$$D_c = \begin{bmatrix} k_{p,1}\left(a+\frac{T}{2T_{i,1}}\right) & 0 \; \dots \\ 0 & k_{p,2}\left(a+\frac{T}{2T_{i,2}}\right) \; \dots \\ -k_{p,1}\left(1+\frac{T}{2T_{i,2}}\right) & 0 \\ 0 & -k_{p,2}\left(1+\frac{T}{2T_{i,2}}\right) \end{bmatrix}.$$

Applying the proposed synthesis procedure to compute multiloop I-P controllers, $\chi_1 = k_{p,1}$, $\chi_2 = T/T_{i,1}$, $\chi_3 = k_{p,2}$, and $\chi_4 = T/T_{i,2}$, and the decoupling precompensator with the following structure:

$$A_d = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \chi_5 & \chi_6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \chi_7 & \chi_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \chi_9 & \chi_{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \chi_{11} & \chi_{12} \end{bmatrix},$$

$$B_d^T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$C_d = \begin{bmatrix} \chi_{13} & \chi_{14} & \chi_{15} & \chi_{16} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \chi_{17} & \chi_{18} & \chi_{19} & \chi_{20} \end{bmatrix},$$

it is achieved $k_{p,1} = 0.0069$, $T_{i,1} = 42.9311$, $k_{p,2} = 0.0118$, $T_{i,2} = 139.6382$, and the precompensator with the following transfer functions:

$$D_{1,1}(z) = \frac{44.101(z-0.8185)}{(z^2-1.536z+0.9748)},$$

$$D_{1,2}(z) = \frac{-157.33(z-0.9592)}{(z^2-1.466z+0.8945)},$$

$$D_{2,1}(z) = \frac{-88.607(z-0.9681)}{(z-0.381)(z-0.5739)},$$

$$D_{2,2}(z) = \frac{112.54(z-0.8992)}{(z^2-0.7721z+0.2231)}.$$

This decoupling control results in the guaranteed approximation error of $\max_{\alpha\in\Omega} \|E(z)\|_\infty = 0.1029$. First-order precompensator results poor decoupling for this case.

To verify the achieved robust decoupling control on the interactions among control loops, the system is simulated with step changes in the reference signals $r_1(t) = \mathbf{1}(t)$, $r_2(t) = \mathbf{1}(t-3000)$, where $\mathbf{1}(t-\tau)$ is the unit-step function translated by τ. The transient responses of the plant outputs and manipulated variables are presented in Fig. 3 and Fig. 4, respectively. It is noticeable in Fig. 3 that, because of the choice of the reference model with the diagonal structure, the robust decoupling controller improves significantly the decoupling among the control loops of the system. Since the model matching error is low, both tracking responses are similar to the specified reference model responses for the 4 polytope vertices. Despite the time delay introduced in the transient responses, it can be considered that both tracking responses are improved in relation to previous results in the literature [15], [16]. The Fig. 4 shows that is not necessary a higher control effort to achieve the tracking response improvements. There is a trade-off in the choice of the reference model. Reference models with faster transient responses result in higher approximation errors and consequently higher interactions among control loops. The interactions can be further reduced choosing reference models with slower transient responses.

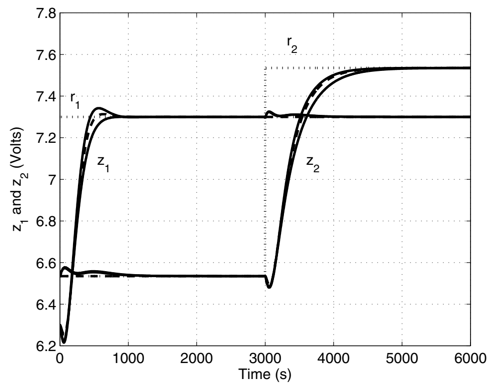

Fig. 3. Transient responses of the lower tank levels (solid line), reference model outputs (dashed line), and set-point signals (dotted) for the 4 polytope vertices.

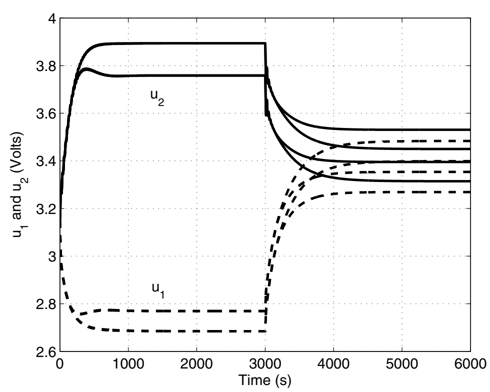

Fig. 4. Transient responses of the pump voltages u_1 (dashed line) and u_2 (solid line) for the 4 polytope vertices.

To demonstrate that the necessity of the decoupling precompensator, the proposed procedure is applied to compute just the multiloop PI controllers without the decoupling precompensator. It is achieved $k_{p,1} = 0.1584$, $T_{i,1} = 57.5836$, $k_{p,2} = 1.1252$, and $T_{i,2} = 359.6777$, resulting the guaranteed approximation error of $\max_{\alpha\in\Omega} \|E(z)\|_\infty = 0.4083$. This higher approximation error results in worst decoupling as illustrated in Fig. 5.

Similar results can be achieved considering a simplified decoupler configuration where its diagonal elements are set as unity, $D_{1,1}(z) = 1$ and $D_{2,2}(z) = 1$:

$$D(z) = \left[\begin{array}{cccc|cc} 0 & 1 & 0 & 0 & 0 & 0 \\ \chi_5 & \chi_6 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \chi_7 & \chi_8 & 1 & 0 \\ \hline \chi_9 & \chi_{10} & 0 & 0 & 1 & 0 \\ 0 & 0 & \chi_{11} & \chi_{12} & 0 & 1 \end{array}\right].$$

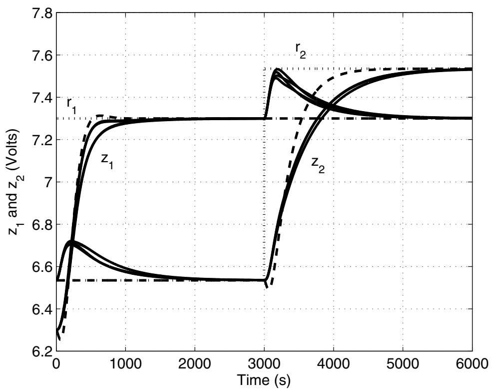

Fig. 5. Transient responses of the lower tank levels (solid line), reference model outputs (dashed line), and set-point signals (dotted) for the 4 polytope vertices.

The multiloop PI controllers with $k_{p,1} = 0.1651$, $T_{i,1} = 55.5231$, $k_{p,2} = 0.4023$, and $T_{i,2} = 174.2660$, and the decoupling precompensator:

$$D_{1,2} = \frac{-5.3269(z-0.9455)}{(z^2-1.145z+0.6322)},$$

$$D_{2,1}(z) = \frac{-2.924(z-0.9632)}{(z-0.3106)(z-0.7316)},$$

result in $\max_{\alpha\in\Omega} \|E(z)\|_\infty = 0.1199$ and the transient responses presented in Fig. 6. The decoupling and tracking responses are similar to the more complex decoupling control configuration.

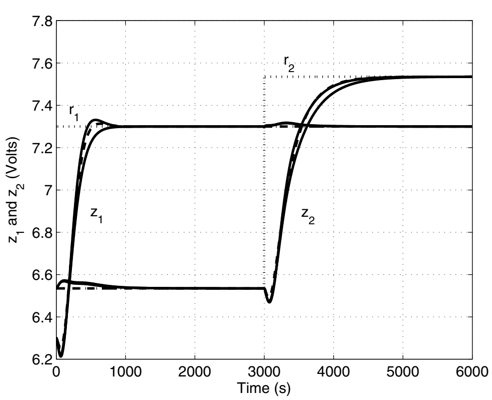

Fig. 6. Transient responses of the lower tank levels (solid line), reference model outputs (dashed line), and set-point signals (dotted) for the 4 polytope vertices.

V. Conclusions

A new robust decoupling control synthesis procedure for uncertain discrete-time linear time-invariant multivariable systems was proposed here. Decoupling of the multivariable system and tracking response performance are considered as control objectives. It was verified that the reference model approximation strategy can assure satisfactory decoupling and tracking response performance. The proposed procedure was illustrated by an application to a non-minimum phase operating point of a quadruple-tank process. The advantages of the proposed synthesis procedure are to consider uncertain systems and to compute simultaneously the multiloop PI/PID controllers and suboptimal physically realizable decouplers.

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