Probabilistic DC Load-Flow Based on Two-Point Estimation (T-PE) Method

G. Mokhtari, *Student Member, IEEE*, A. Rahiminezhad, A. Behnood, J. Ebrahimi, *Student Member,* IEEE, G. B. Gharehpetian*, Senior Member, IEEE*

***Abstract*- In this paper, a Probabilistic DC Load Flow based on Two-Point Estimation (T-PE) Method is proposed. Many PLF methods have been used to study load flow uncertainties. But the calculation time of these methods are considerable. The proposed method uses only two calculations of the deterministic load flow, to calculate the statistical moments of the load-flow solution. Therefore, the proposed method is faster than other methods. In this study, it is assumed that the probabilistic density function of bus injections, probability of lines outage and probability of unit outages can be estimated or measured. The proposed method has been applied to IEEE 14-bus and IEEE 30-bus test systems. The results indicate that the proposed method reduces the calculation time while maintaining sufficient accuracy.**

***Index Terms*- Branch outage, Cumulative Distribution Function (CDF), DC load flow, Generation Shift Factor(GSF), Statistical moment, T-PEM, Uncertainty, Unit outage.**

I. INTRODUCTION

In open access systems, the value of loads, generations and network topology are not deterministic variables. Therefore, the probabilistic load flow should be used to identify network conditions.

In long term transmission system expansion planning, main problem is finding proper location of transmission lines. So, probabilistic DC-load flow is used in more applications. Many PLF methods have been proposed to model load flow uncertainties. The Monte-Carlo simulation determines the Probabilistic Density Function (PDF) of the state vector and lines flow by repeating the deterministic load flow. The computation burden makes this process unattractive [1]. The convolution technique is another method to obtain the PDF of line flows [2-5]. In this method, input variables must be independent and the computation time is not suitable [6]. In [1], the power flow of lines calculated based on Cumulants and Gram-Charlier (CG-C) method and compared with Monte Carlo method. The accuracy is very high and the same as Monte Carlo method. The result of this method is acceptable if uncertain parameters are independent. In addition, CG-C method can only be used to approximate the Cumulative Distribution Function (CDF) of random variable [2]. As a result, there is a need for a accurate method with relatively low calculations time.

In this paper, the probabilistic DC-load flow has been used based on T-PE Method. The method has been applied to IEEE14-bus and 30-bus test systems to show the effectiveness of the proposed.

II. PROBABILISTIC DC- LOAD FLOW

The load flow study computes the steady-state solution of the power system by using equations (1) and (2).

$$P_i = f_i(\delta_1,........,\delta_n,V_1,..........,V_n) \tag{1}$$

$$Q_i = g_i(\delta_1,........,\delta_n,V_1,..........,V_n) \tag{2}$$

In PLF studies, the input variables P_i and Q_i are defined by probabilistic density functions. It is preferable to apply linear approximation to equations (1) and (2), so that the state variables could be solved as a linear combination of input variables. In long term power system planning, the main problem is to locate transmission and generation facilities in the appropriate places and in time, to satisfy the customer's real power demand. Based on these considerations, this paper uses DC load flow in the formulation of PLF problems.

To determine a linear approximation of line flows as a function of input variables (load, generation and network topology), the following parameters should be determined.

A. Generation Shift Factor

Based on Wood and Wollenbergs method [7], the coefficient of the linear relationship between incremental flow on line j (connecting bus a to b) and injected active power of bus m called Generation Shift Factor ($GSF_{j,m}$) and express by the following equation.

$$GSF_{j,m} = \frac{X_{am} - X_{bm}}{x_{ab}} \tag{3}$$

G. Mokhtari is an MSc student in the Electrical Engineering Department, Amirkabir University of Technology (e-mail: gh_mo62@aut.ac.ir).

A. Rahiminezhad is an MSc student in the Electrical Engineering Department, Amirkabir University of Technology (e-mail: abrh328@yahoo.com).

A. Behnood is an MSc student in the Electrical Engineering Department, Amirkabir University of Technology (e-mail: aref.behnood@aut.ac.ir).

J. Ebrahimi is an MSc student in the Electrical Engineering Department, Amirkabir University of Technology (e-mail: j_ebrahimi@aut.ac.ir).

G. B. Gharehpetian is with the Electrical Engineering Department, Amirkabir University of Technology (e-mail: grpetian@aut.ac.ir).

978-1-4244-7128-7/10/$26.00 ©2010 IEEE

Where, x_{ab} is the reactance of the transmission line connecting bus a to bus b, X_{am} is the element on the a-th row and the m-th column of the bus reactance matrix, X.
Therefore, the line j power flow from bus a to bus b (P_j), can be expressed by the following equation:

$$P_j = \sum_m GSF_{j,m} . P_m \tag{4}$$

B. Line Outage Shift Factor

The outage of line k can result in the power flow changes on line j, which can be written, as follows:

$$\Delta P_{j,k} = LOSF_{j,k} . P_k \tag{5}$$

where, P_k is the power flowing on the line k before its outage.

The $LOSF$ is the value of redistribution that can be computed using the following equation [2]:

$$LOSF_{j,k} = \frac{x_{rs}(X_{ar} - X_{as} - X_{br} + X_{bs}}{x_{ab}(x_{rs} - (X_{rr} + X_{ss} - 2X_{rs}))} \tag{6}$$

Using equations (4) and (6), $\Delta P_{j,k}$ can be rewritten as follows:

$$\Delta P_{j,k} = LOSF_{j,k} . \sum_m GSF_{k,m} . P_m \tag{7}$$

C. Line Uncertainty Modeling

Using conditional probability model, the uncertainty of the line outage can be model by the following equation:

$$P_j \approx P_j^{nor} . \rho^{nor} + \sum_k P_j^{lk} . \rho^{lk} \tag{8}$$

where, P_j^{nor} is the power flow on the line j under normal condition. ρ^{nor} is the probability of the normal condition. P_j^{lk} is the power flow on the j-th transmission line when the outage happens on the k-th line. P^{lk} is the line k-th outage probability.

Therefore an approximation of line flow, P_j considering network topology uncertainties can be expressed by the following equation:

$$P_j = \sum_m h_{j,m} . P_m \tag{9}$$

where,

$$h_{j,m} = GSF_{j,m} . \rho^{nor} + \sum (GSF_{j,m} + LOSF_{j,k} . GSF_{k,m}) . \rho^{lk} \tag{10}$$

D. Unit Uncertainty Modeling

To model the unit outage in this paper, the binomial distribution has been used to determine its capacity, considering the outage probability of the unit.

III. TWO- POINT-ESTIMATION (T-PE) METHOD

In the probabilistic load flow methods, in order to model the uncertainties of the injected power of generators, loads and the network topology, the probabilistic moment of the output variables should be determined. the Calculation time and its accuracy is very important. T-PE method is accurate and fast enough for the calculation of the probabilistic moment of output variables [4].

The T-PE method can be used to calculate the statistical moments of a random quantity which is a function of one or several random variables [4].

In order to efficiently and accurately estimate the uncertainty involved in the load flow calculations, a T-PE probabilistic load flow algorithm similar to that presented in [4], is used in this paper.

Using equation (9), the power flow on the j-th line can be written as a function of input variables (generations and loads in all buses).

$$P_j = f(G_1, \ldots, G_N, D_1, \ldots D_N) \tag{11}$$

Let z_i be the input variable with the probability density function f_{zi}.

$$P_j = f(z_1, \ldots, z_i, \ldots, z_N, z_{N+1}, \ldots, z_{2N}) \tag{12}$$

In the proposed method, two values of z_i, i.e., $z_{i,1}$ and $z_{i,2}$, defined by the equation (13), have been used, to replace f_{zi} by the first three moments of f_{zi} [4].

$$z_{i,k} = \mu_{zi} + \xi_{i,k} \times \sigma_{zi} \tag{13}$$

where, μ_{zi} is the mean and standard deviation of f_{zi}, σ_{zi} is the standard deviation of f_{zi} and $\xi_{i,k}$ is expressed by the following equations:

$$\xi_{i,k} = \frac{\lambda_{i,3}}{2} + (-1)^{3-k} \times \sqrt{m + (\frac{\lambda_{i,3}}{2})^2} \tag{14}$$

Where, $\lambda_{i,3}$ is the coefficient of skewness of z_i which can be calculated, as follows:

$$\lambda_{i,3} = \frac{E[(zi - \mu_{xi})^3]}{(\sigma_{zi})^3} \tag{15}$$

The value of the $z_{i,1}$ and $z_{i,2}$ are used to produce two estimates of the P_j, *i. e.,* $P_j(i,1)$ and $P_j(i,2)$, as follows:

$$P_j(i,k) = f(\mu_{z1}, \ldots, z_{i.k}, \mu_{z(N+1)}, \ldots, \mu_{z(2N)}) \tag{16}$$

The equation (17) presents $w_{i,k}$, the weighting factor for the point $(\mu_{x1}, \ldots, xi,k, \ldots, \mu_{xN}, \mu_{x(N+1)}, \ldots, \mu_{x(2N)})$ and used to scale the estimates considering the skewness of the probability distribution of P_j.

$$w_{i,k} = \frac{1}{m} \times (-1)^k \times \frac{\xi_{i,3-k}}{\zeta_i} \tag{17}$$

where,

$$\zeta_i = 2 \times \sqrt{m + (\frac{\lambda_{i,3}}{2})^2} \tag{18}$$

The p-th moment of P_j can be determined by the proposed method based on the equation (19).

$$E(P_j^{\,p}) \cong \sum_{i=1}^{m} \sum_{k=1}^{2} w_{i,k} \times [P_j(i,k)]^p \qquad (19)$$

Now, the standard deviation of the P_j can be calculated by the equation (20).

$$\sigma_{P_j} = \sqrt{\mathrm{var}(P_j)} = \sqrt{E(P_j^{\,2}) - (E(P_j))^2} \qquad (20)$$

The algorithm of the proposed method known as T-PE method is shown in Fig. 1.

IV. Simulation Results

The proposed method has been applied to the IEEE 14-bus and 30-bus test systems.

To use the proposed algorithm shown in Fig. 1. All loads are assumed to be normally distributed, their mean values are equal to the base case bus loads and their standard deviations are equal to 10% of their mean values. To model generation uncertainties, the Forced Outage Rate (FOR) of each generation units are set to 0.1. The capacity can be determined by using the binomial distribution. To model transmission uncertainties, FORs of each lines are set to 0.2%.

A. IEEE 14-bus

To estimate the mean and standard deviation of power flows on the transmission lines, these methods have been compared in this paper. Table I and II list the results of estimation of these parameters by using Monte-Carlo, CG-C and the proposed method known as T-PE method.

Monte Carlo simulation should repeat the process of deterministic load flow in each simulation. The number of trial using in this method is set to 5000.

In CG-C method [1], 6 orders for the approximation of the CDF have been used. This method have been compared wit Monte-Carlo simulation in [2].

In this paper, the results have been compared with Monte-Carlo simulation and CG-C methods. Considering the Tables I and II, it can be said that the proposed method has a enough accuracy. For some cases, the T-PE solution is near to the Monte-Carlo simulation and in some other cases CG-C solution is near to the Monte Carlo simulation.

to compare the accuracy of the purposed method with CG-C method, Average Root Mean Square (*ARMS*) error is computed by using 5000 iteration results of the Monte Carlo method. ARMS is expressed by the following equation:

$$ARMS = \frac{\sqrt{\sum_{i=1}^{N} (CG_i - MC_i)^2}}{N} \qquad (21)$$

Where, *CGi* is the *i*-th point value on the cumulative

distribution curve calculated by using the CG-C and T-PE methods, *MCi* is the *i*-th point value on the cumulative distribution curve calculated by using the Monte-Carlo method and N is the number of points.

Table III gives *ARMS* parameter calculation for both methods. In the case of approximation by CG-C method. the results are better than the proposed method.

B. IEEE 30-bus

Fig. 1 and 2, indicate the CDF of the line power flow between buses 1 and 2 and buses 21 and 22. Comparing these figures with the result of CG-C method and Monte-Carlo method, presented in [2], it can be said that the proposed method has enough accuracy to estimate CDF.

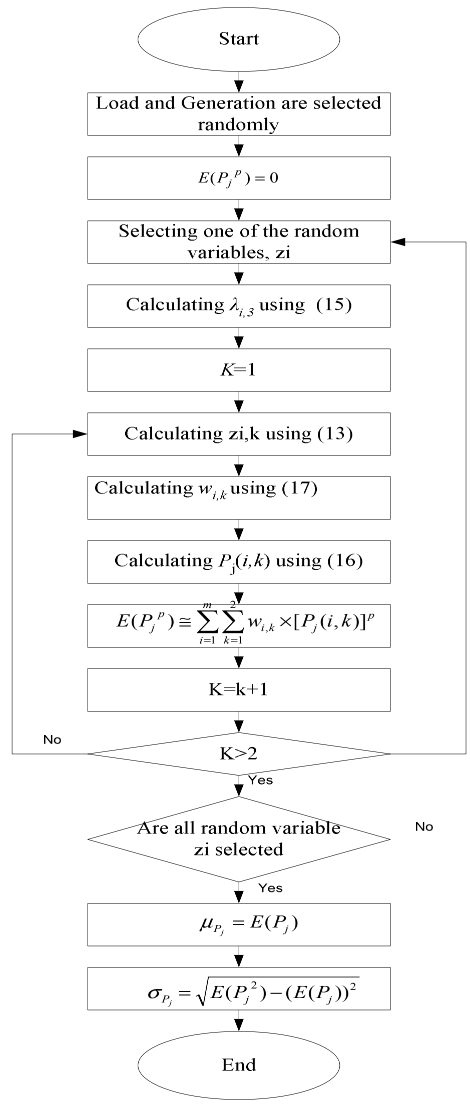

Fig. 1.The proposed method

TABLE I
MEAN OF POWER FLOW ON IMPORTANT LINES

	Monte-Carlo	CG-C	T-PE
Line 2-3	70.83	73.01	72.624
Line 4-7	24.43	30.91	30.062
Line 2-4	52.914	57.396	57.015
Line 2-5	38.394	42.393	41.862
Line 3-4	27.261	25.123	25.343
Line 4-5	62.604	64.840	65.432
Line 4-7	24.460	30.077	30.062
Line 4-9	14.099	17.337	17.329
Line 5-6	37.172	43.775	43.815
Line 1-5	69.265	74.129	75.289

TABLE II
STANDARD DEVIATION OF POWER FLOW ON IMPORTANT LINES

	Monte-Carlo	CG-C	T-PE
Line 2-3	5.266	5.300	6.356
Line 4-7	1.510	1.620	1.129
Line 2-4	2.358	2.392	1.327
Line 2-5	1.521	1.529	1.796
Line 3-4	4.657	4.683	4.658
Line 4-5	4.005	4.021	4.090
Line 4-7	1.510	1.620	1.558
Line 4-9	0.871	0.933	0.776
Line 5-6	1.561	1.703	1.603
Line 1-5	3.304	3.290	3.752

TABLE III
ARMS FOR LINE 1-2

	CG-C(6th)	*T-PE*
ARMS	.39%	.44%

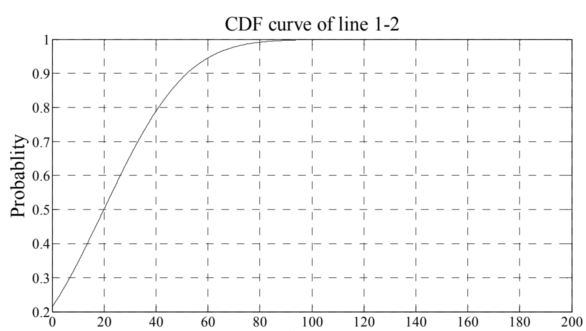

Fig. 2. Cumulative Distribution Curve of Line 1-2

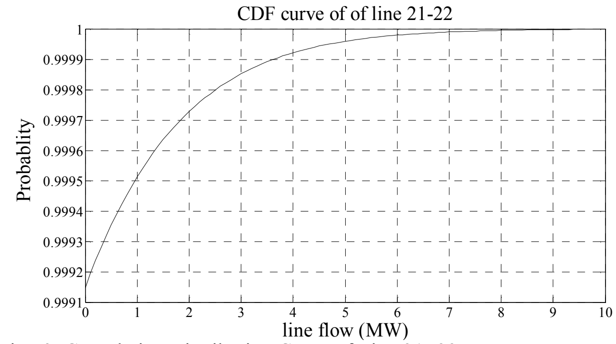

Fig. 3. Cumulative Distribution Curve of Line 21- 22

Table IV lists the calculation time used by different methods. All calculations are carried out on a Pentium IV 3.2-GHz PC, by using a program developed in MatLab. It can be seen that T-PE method is faster than other methods.

TABLE IV
CALCULATION TIME OF METHODS

	Monte-Carlo (5000 trial)	*CG-C (6th)*	*T-PE*
IEEE-14buses	6.54S	2.34S	1.54S
IEEEE-30buses	16.97S	4.73S	3.06S

V. CONCLUSION

In this paper, to decrease the calculation time of the probabilistic load flow with considering of unit and branch outage, the Two Point Estimation (T-PE) has been proposed. in this method, uncertainties of generations, loads and network topology have been modeled in the injected power of buses. Then, the line power flows can be determined as a function of the injected powers. To study the proposed method, it has been applied to test system. The results have been compared with Monte-Carlo and CG-C method. it is shown that this method is accurate enough and it is faster than other methods.

VI. REFERENCES

[1] P. Zhang and S. T. Lee, "Probabilistic load flow computation using the method of combined Comulants and Gram-Charlier expansion," IEEE Trans. on Power Systems, Vol. 19, No. 1, pp. 676-682, Feb. 2004.

[2] L. Min, P. Zhang, "A Probabilistic Load Flow with Consideration of Network Topology Uncertainties," proc. IEEE, ISAP2007, PP.1-5, Nov. 2007.

[3] L. Min, P. Zhang, "A Probabilistic Load Flow with Consideration of Network Topology Uncertainties," proc. IEEE, ISAP2007, PP.1-5, Nov. 2007.

[4] Z. Hu, X. Wang, "A Probabilistic Load Flow Method Considering Branch Outages," IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 21, NO. 2, pp.507-514, MAY 2006.

[5] M. Davari, F. Toorani, H. Nafisi, M.. Abedi, G.B. Gharehpetian, "Determination of Mean and Variance of LMP Using Probabilistic DCOPF and T-PEM," *Proc. IEEE,* PECon08, pp. 1280-1283, 2008.

[6] B. Borkowska, "Probabilistic load flow," *IEEE Trans. Power App. Syst*, vol. PAS-93, pp. 752–759, Apr. 1974.